Clarifying Inflation Models for and from CMB Observations + LSS

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CONCORDANCE COSMOLOGICAL MODEL :

STANDARD COSMOLOGY TODAY

$$\Lambda CDM \left\{ \begin{array}{l} \text{COLD} \\ \text{DARK} \\ \text{MATTER} \\ + \Lambda > 0 \end{array} \right.$$

EXPLAINS THE OBSERVATIONS:

THREE YEARS WMAP DATA

- SMALL SCALE CMB DATA (SMALL angle, HIGH L)
- LIGHT ELEMENTS ABUNDANCES
- LARGE SCALE STRUCTURE (LSS) OBSERVATIONS
- SUPERNOVAE LUMINOSITY/DISTANCE RELATIONSHIP
- LYMAN α FOREST OBSERVATIONS
- GRAVITATIONAL LENSING OBSERVATIONS
- H_0 (HUBBLE CONSTANT) MEASUREMENTS (HST)
- PROPERTIES OF CLUSTERS OF GALAXIES

⋮

PRESENT UNDERSTANDING

INFLATION IS PART OF THE CONCORDANCE (STANDARD) MODEL OF THE UNIVERSE

- THE UNIVERSE STARTS BY AN INFLATIONARY ERA OF INFLATION (ACCELERATED EXPANSION $\frac{d^2a}{dt^2} > 0$)
- DURING THE INFLATIONARY ERA THE UNIVERSE EXPANDED BY AT LEAST SYXTY EFOLDS $e^{60} \simeq 10^{26}$.
- INFLATION LEASTS $\simeq 10^{-34}$ sec.
- INFLATION ENERGY SCALE $\sim 10^{16}$ GeV : GRAND UNIFICATION SCALE
- FOUR DIMENSIONS
- SEMICLASSICAL THEORY OF GRAVITY : (GR+QFT) GENERAL RELATIVITY + QUANTUM FIELD THEORY
- EFFECTIVE THEORY

NEW ERA OF INFLATION AFTER WMAP

Clarifying Inflation Models

-Effective Theory

-*Precise Inflaton Potential* from Effective Field Theory and the WMAP data

-*Quantum Corrections:* to the Inflaton Potential, to the Primordial Power

-NO fine tuning

Inflation as known today should be considered as an Effective Theory

That is, is Not a fundamental theory but a theory on a condensate (the inflaton field) which follows from a more fundamental theory (the GUT model)

The Inflaton field may NOT correspond to any real particle (even unstable) but is just an

effective description while the microscopic derivation should come from the GUT

Inflation is to the microscopic GUT theory,

like the effective Ginsburg-Landau theory of superconductivity is to the microscopic BSC theory,

or like the O(4) sigma model is to QCD

To provide a clear understanding of Inflation

and the Inflaton Potential from

Effective Field Theory and the WMAP data

(2)

GRAND UNIFICATION SCALE

1015 GeV $\leq$ E $\leq$ 1016 GeV

Three experimental supports:

(1) Unification of couplings

(2) Neutrino Oscillations

(3) Inflation

This clearly places Inflation within the perspective and understanding of effective theories in particular physics, and sets up a clean way to directly confronts the inflationary predictions

with the forthcoming CMB data and select a definitive model

IMPLICATIONS FOR GRAND UNIFICATION:

GRAND UNIFICATION SCALE:
Three experimental supports:

(1) Unification of couplings in the Standard Model with the Renormalization group For the Standard Model, couplings get unified approximately at E $\sim$ 1016 GeV.

(2) Neutrino Oscillations: and Neutrino masses currently explained by the See-Saw mechanism

$$\boxed{\Delta m_\nu \sim M^2_{FERMI} / M}$$

M_{Fermi} $\sim$ 250 GeV, $M >> M_{Fermi}$ and Δm_ν is the difference of neutrino masses for the different flavors.

- *The observed values for $\Delta m_\nu \sim$ 0.009-0.05 eV naturally call for a mass scale M close to the GUT scale: $M \sim 10^{15\text{-}16}$ GeV*
- *The inflaton potencial relation $V(\phi) = M^4 v\left(\frac{\phi}{M_{Pl}}\right)$*

Ressembles the moduli potential from supersymmetry breaking: $v_{susy}(\phi) = m^4_{susy}\, v(\Phi/M_{Pl})$

- *Our approach, combined with Δm_ν Implies ms ~ 1016 GeV The SUSY breaking scale ms is at the GUT scale $\boxed{ms \sim mGUT}$*

(3) Inflation

- *We find that the mass scale of the inflaton 1013 GeV can be related with M_{GUT} by a see-saw relation*

$$m \simeq \frac{M^2_{GUT}}{M_{Planck}}$$

- *The Inflaton describes a condensate in a GUT theory (fermion-antifermion) pairs.*
- *There is no solid basis to identify such a condensate field with a given fundamental field in a SUSY or SUGRA model.*
- *Moreover, the number of susy models is so large: there is no way to predict the which is THE correct model.*

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IMPLICATIONS FOR STRING THEORY

- *To generates Inflation Needs first to generate a mass scale like the inflaton mass m.*
- *Such scale is NOT present in the string action neither in the effective fields (dilaton, graviton, antisymmetric tensor).*
- *Without the presence of the mass scale m and M_{GUT}, there is NO hope in string theory to describe a correct inflationary cosmology describing the CMB fluctuations.*
- *Such scale should be generated dynamically perhaps from the string vacuum, but this is still an open problem far from being solved.*
- *Since No microscopic derivation of inflation from a GUT model is available so far, it would seem too ambitious at this stage to look for a microscopic derivation of Inflation from string theory.*
- *The derivation of Inflation reproducing observed CMB fluctuations is at present too hard.*
- *An effective description of Inflation in String theory (string matter plus background) could be at reach*
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- **The Effective Field Theory (EFT) Approach relies on the *separation* between the energy scale of inflation** and *the higher energy scale of the earlier stage (cutoff scale)* which here is the Planck scale.

- **Scale of inflation**: **Hubble parameter** during the relevant stage of inflation (wavelengths of cosmological relevance cross the horizon).

- **EFT expansion: defined by the dimensionless ratio** $H(\Phi_0)/M_{PL}$.

- **Reliability of the expansion : improves** upon **dynamical evolution** since the scale of inflation diminishes with time.

- **EFT expansion** is an **excellent** one since the amplitudes of tensor and scalar perturbations Δ_T and Δ_R respectively are given

$$\Delta_T = \frac{\sqrt{2}\,H}{\pi\,M_{PL}} \quad , \quad \frac{H}{M_{PL}} = 2\,\pi\,\Delta_R\,\sqrt{2}\,\varepsilon_v$$

$\varepsilon_v << 1$. ● **WMAP data** $\Delta_R = 0.47 \times 10^{-4}$ provide **strong observational support to the validity of an effective field theory for inflation well below the Planck scale and to the** (H/M_{PL}) **expansion.**

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effective potential to be

QUANTUM CORRECTIONS

$$V_{eff}(\Phi_0) = V(\Phi_0)\left[1 + \frac{H_0^2}{3\,(4\pi)^2\,M_{Pl}^2}\left(\frac{\eta_v - 4\,\epsilon_v}{\eta_v - 3\,\epsilon_v} + \frac{3\,\eta_\sigma}{\eta_\sigma - \epsilon_v} + \mathcal{T}\right)\right]$$

where $V(\Phi_0)$ is the *classical* inflaton potential, $\eta_v, \epsilon_v, \eta_\sigma$ slow-roll parameters and $\mathcal{T} = \mathcal{T}_\Phi + \mathcal{T}_s + \mathcal{T}_t$ -145.15 is the total trace anomaly from the scalar metric, tensor, light scalar and fermion contribu

The terms that feature ratios of slow roll parameters arise from superhorizon contributions from cur field perturbations. The last term in eq.(1) is independent of slow-roll parameters and is complete the trace anomalies of the different fields. It is the hallmark of the subhorizon contributions.

In the case when the mass of the light bosonic scalar field is much smaller than the mass of the infl we find the following result for the scalar curvature and tensor fluctuations including the one-loop qua

$$|\Delta^{(S)}_{k,eff}|^2 = |\Delta^{(S)}_k|^2\left\{1 + \frac{2}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[1 + \frac{\frac{3}{8}\,r\,(n_s-1) + 2\,\frac{dn_s}{d\ln k}}{(n_s-1)^2} + \frac{2903}{40}\right]\right\}$$

$$|\Delta^{(T)}_{k,eff}|^2 = |\Delta^{(T)}_k|^2\left\{1 - \frac{1}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[-1 + \frac{1}{8}\,\frac{r}{n_s-1} + \frac{2903}{20}\right]\right\}\,,\quad \text{UV}$$

$$r_{eff} \equiv \frac{|\Delta^{(T)}_{k,eff}|^2}{|\Delta^{(S)}_{k,eff}|^2} = r\,\left\{1 - \frac{1}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[1 + \frac{\frac{3}{8}\,r\,(n_s-1) + \frac{dn_s}{d\ln k}}{(n_s-1)^2} + \frac{8709}{20}\right]\right\}$$

The quantum corrections turn out to **enhance** the scalar curvature fluctuations and to **reduce** the t as well as their ratio r. The quantum corrections are always small, of the order $\left(\frac{H_0}{M_{Pl}}\right)^2$, but it is

Expected CMB constraints on Δ_T should still improve this support.

- **Δ_T *and* Δ_R expressed in terms of the *semiclassical and quantum gravity temperature scales***

$$T_{sem} = \frac{\hbar\,H}{2\,\pi\,kB} \qquad T_{PL} = \frac{M_{PL}\,c^2}{2\,\pi\,kB}$$

- **T_{sem} is the Hawking-Gibbons temperature** of the initial state (Bunch-Davis vacuum) of inflation. **T_{PL} is the Planck temperature 10^{32}° K.**

$$\Delta_T = \frac{\sqrt{2}}{\pi}\,\frac{T_{sem}}{T_{PL}} \qquad \frac{T_{sem}}{T_{PL}} = 2\,\pi\,\Delta_R\,\sqrt{2\varepsilon_v}$$

Therefore, WMAP data yield for the Hawking-Gibbons temperature of inflation:

$$T_{sem} \approx \sqrt{\varepsilon_v}\,10^{28}\ \text{°K}$$

Constructing and Understanding the Precise Inflationary Model from Effective Field theory and the WMAP data

- The *Wilkinson Microwave Anisotropy Probe* (WMAP) has provided a full-sky map of the temperature fluctuations of the CMB
- *Inflation* provides a natural mechanism for the generation of scalar density fluctuations that seed large scale structure, explaining the origin of temperature anisotropy in the CMB
- Wavelength that are of cosmological relevance today re-enter the horizon during the matter dominated era when the scalar (curvature) perturbations induce temperature *anisotropies* that are imprinted in the CMB
- Generic inflationary models predict gaussian adiabatic perturbations with a spectrum that is almost *scale invariant*
- Inflationary dynamics is typically studied by treating the *inflaton* as an homogeneous *classical scalar field* whose evolution is determined by a classical equation of motion. The inflaton quantum fluctuations provide the seeds for the scalar density fluctuations of the metric
- Important aspects of the inflationary dynamics, as the resonant particle production and the non linear back-reaction that it generates, requires a full *quantum* treatment for their consistent description

CONCLUSIONS

- ***Setting $m = 0$ in polynomial potentials implies a <u>non-generic</u> choice. WAMP disfavors such a choice and supports a generic quartic polynomial potential***

$\Downarrow$

A purely quartic potential is disfavored $\Rightarrow$ ***<u>Lower Bound for m: $m > 10^{13}$ GeV</u>***

- ***Spectral Indices data*** $\Rightarrow$ ***should help soon to make a clear selection between inflationary models***
- ***A measured $r < 0.16$*** $\Rightarrow$ ***<u>excludes Chaotic Inflation</u>***
- ***n_s value above or below unit*** $\Rightarrow$ ***exclude either New or Hybrid inflation respectively***
- ***Our approach, combined with*** $\Delta m_\nu \sim \frac{M^2_{Fermi}}{M}$ $\Rightarrow$ ***Implies $m_s \sim 10^{16}$ GeV***

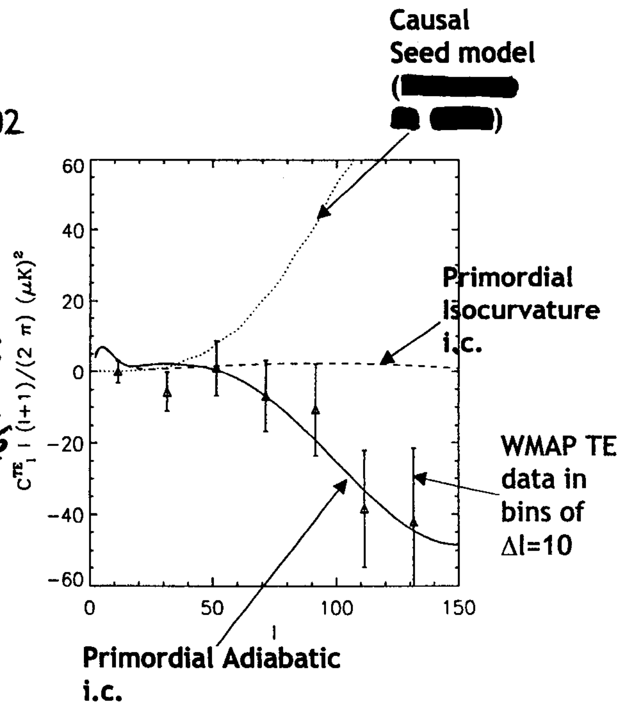
$\Downarrow$

The SUSY breaking scale m_s $\Rightarrow$ ***is at the GUT scale $m_s \sim m_{GUT}$***

Then the mass scale of the inflaton 10^{13} Gev is related $\Rightarrow$ $m \sim \frac{M^2_{GUT}}{M_{Pl}}$

WMAP Supports Single Field Inflationary Models

- Flat universe: $\Omega_{tot} = 1.02 \pm 0.02$
- Gaussianity: $-58 < f_{NL} < 134$
- Power Spectrum spectral index nearly scale-invariant:
  $n_s = 0.99 \pm 0.04$ (*WMAP* only) 2003
  $n_s = 0.95 \pm 0.02$ (WMAP, 2dF) 2006
- Adiabatic initial conditions
- Superhorizon fluctuations (TE anticorrelations)



primordial power spectrum. The slow-roll approximation ... when modes of cosmological relevance ... $N_{efolds} \sim 50$ is the number of efolds before the end of inflation when modes of cosmological relevance today crossed the Hubble radius.

The observational progress permit to start to discriminate among different inflationary models, placing stringent constraints on them. The upper bound on the ratio r of tensor to scalar fluctuations obtained by WMAP [4, 5] rules out the massless ϕ^4 model and **necessarily** implies the presence of a **mass term** in the inflaton potential [5, 9].

Besides its simplicity, the trinomial potential is a physically well motivated potential for inflation in the grounds of the Ginsburg-Landau approach to effective field theories (see for example ref.[11]). This potential is rich enough to describe the physics of inflation and accurately reproduce the WMAP data [4, 5].

The slow-roll expansion plus the WMAP data constraints the inflaton potential to have the form [9]

$$V(\phi) = N_{efolds}\, M^4\, w(\chi)\ , \tag{1.1}$$

where ϕ is the inflaton field, χ is a dimensionless, slowly varying field

$$\chi \equiv \frac{\phi}{\sqrt{N_{efolds}}\, M_{Pl}}\ , \tag{1.2}$$

$w(\chi) \sim \mathcal{O}(1)$ and M is the energy scale of inflation which is determined by the amplitude of the scalar adiabatic fluctuations [4] to be

$$M \sim 0.00319\, M_{Pl} = 0.77 \times 10^{16}\text{GeV}\ .$$

Following the spirit of the Ginsburg-Landau theory of phase transitions, the simplest choice is a quartic trinomial for the inflaton potential [9, 10]:

$$w(\chi) = w_0 \pm \frac{1}{2}\,\chi^2 + \frac{h}{3}\,\sqrt{\frac{y}{2}}\,\chi^3 + \frac{y}{32}\,\chi^4\ . \tag{1.3}$$

INFLATION IS AN ATTRACTOR OF THE DYNAMICAL EQS OF MOTION

THE SLOW ROLL REGIME IS AN ATTRACTOR

- **STARTING WITH** $\Phi^2 \sim V(\Phi)$

(**THAT IS, THE KINETIC ENERGY IS OF THE SAME ORDER OF THE POTENTIAL ENERGY,**)

$\Rightarrow$ **THE INFLATON FIELD ALWAYS REACHS THE SLOW LOW REGIME AFTER A FEW NUMBER OF EFOLDS (typically 1 efold)**

SLOW ROLL HAS A LARGE BASIN OF ATTRACTION

$$y^2 \equiv \frac{\dot{\Phi}^2}{2 M_{Pl}^2 H^2} = 3\left[1 - \frac{V(\Phi)}{3 M_{Pl}^2 H^2}\right] = \epsilon_V$$

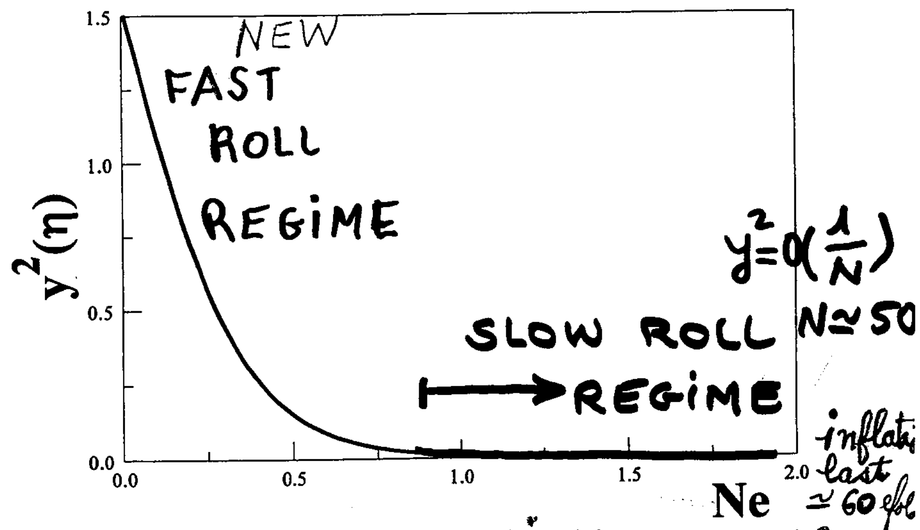
$$= \frac{\dot{\chi}^2}{2 h^2 N} = 3\left[1 - \frac{w(\chi)}{3 h^2}\right], \quad N \sim 50, \quad V(\phi) > 0$$

$$0 \leq y^2 \leq 3$$

$y^2 \simeq O\left(\frac{1}{N}\right) \ll 1$: SLOW ROLL

$y^2 = O(1)$: FAST ROLL

$$\boxed{\dot{\Phi}^2 \simeq V(\Phi) \text{ in FAST ROLL}}$$



NEW INFLATION (small field)

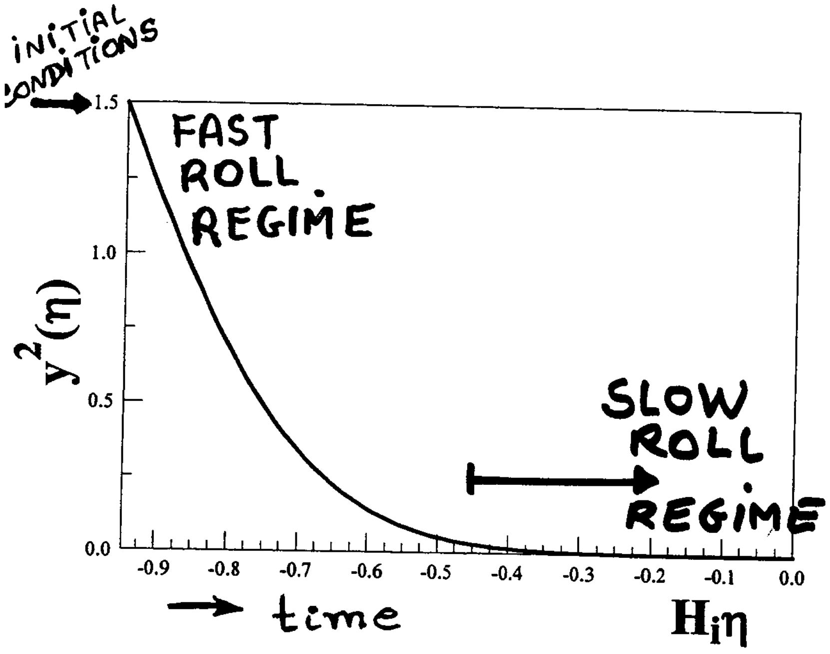
$$V(\Phi) = \frac{\lambda}{4}\left(\Phi^2 - \frac{M^4}{\lambda M_{Pl}^2}\right)^2,$$

$$\boxed{V(\Phi) = N M^4 w(\chi)} \quad \boxed{\chi = \frac{\Phi}{\sqrt{N}\, M_{Pl}}}$$

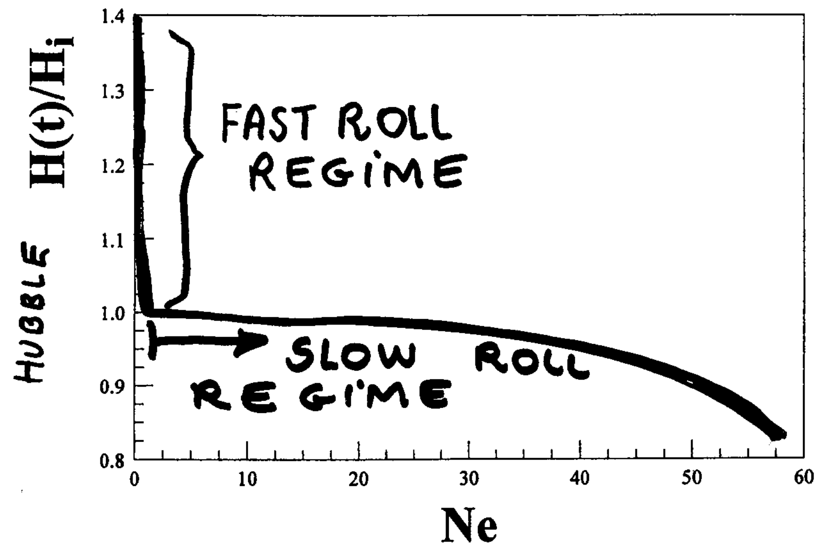
$$w(\chi) = \frac{G}{4}\left(\chi^2 - \frac{1}{G}\right)^2$$

($G = 0.4$ in the plot)

NEW INFLATION



HUBBLE vs Nefolds



$$y^2 \equiv \frac{\dot{\Phi}^2}{2M_{Pl}^2 H^2} = 3\left[1 - \frac{V(\Phi)}{3M_{Pl}^2 H^2}\right] = \frac{\dot{\chi}^2}{2Nh^2}$$

$\boxed{0 \leq y^2 \leq 3}$; $\boxed{y^2 = \epsilon_v = O\left(\frac{1}{N}\right) \text{ in SLOW ROLL}}$

$\boxed{y^2 = O(1) \text{ in FAST ROLL}}$

FAST ROLL TAKES PLACE DURING THE **FIRST EFOLD** ONLY where the coefficients w_0, h and y are dimensionless and of order one and the signs $\pm$ correspond to large and field inflation, respectively (chaotic and new inflation, respectively). Inserting eq.(1.3) in eq.(1.1) yields,

$$V(\phi) = V_0 \pm \frac{m^2}{2}\,\phi^2 + \frac{m\,g}{3}\,\phi^3 + \frac{\lambda}{4}\,\phi^4 \,.$$

where the mass m^2 and the couplings g and λ are given by the following see-saw-like relations,

$$m = \frac{M^2}{M_{Pl}} \quad , \quad g = h\,\sqrt{\frac{y}{2\,N}}\,\left(\frac{M}{M_{Pl}}\right)^2 \quad , \quad \lambda = \frac{y}{8\,N}\,\left(\frac{M}{M_{Pl}}\right)^4 \quad , \quad V_0 = N\,M^4\,w_0 \,.$$

where $N \equiv N_{efolds}$. Notice that $y \sim \mathcal{O}(1) \sim h$ guarantee that $g \sim \mathcal{O}(10^{-6})$ and $\lambda \sim \mathcal{O}(10^{-12})$ without any fine as stressed in ref. [9]. That is, the smallness of the couplings directly follow from the form of the inflaton po eq.(1.1) and the amplitude of the scalar fluctuations that fixes M [9].

The small coupling limit $y \to 0$ of eqs.(1.3)-(1.4) corresponds to a quadratic potential while the strong co limit $y \to \infty$ yields the massless quartic potential. The extreme asymmetric limit $|h| \to \infty$ yields a massive without quadratic term. In such limit the product $|h|\,M^2$ must be kept fixed since it is determined by the am of the scalar fluctuations.

We study here new inflation with the trinomial potential eqs.(1.3)-(1.4) and hybrid inflation [see below], t models fulfill the observational constraints. We compute in both scenarios n_s, r and the running dn_s/d functions of the parameters of the models, derive explicit formulae for n_s, r and $dn_s/d\ln k$ and provide relevan Moreover, we plot the ratio r and the running $dn_s/d\ln k$ as functions of the scalar index n_s. Since the value now known [5]-[8], these plots allow us to predict the values of r and $dn_s/d\ln k$ for the different inflationary considered.

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The three years WMAP data indicate a red tilted spectrum ($n_s < 1$) with a small ratio $r < 0.28$ of te uctuations [5]. More precisely, the three years WMAP data [5] as well as ref. [6] yield

$$n_s = 0.95 \pm 0.02 \,.$$

/e find that for $n_s = 0.95$ and any value of the asymmetry h [see figs. 4 and 5], new inflation with otential eqs.(1.3)-(1.4) predicts

$$\text{trinomial potential new inflation for } n_s = 0.95 : \quad 0.03 < r < 0.04 \quad \text{and} \quad -0.00070 < dn_s/d\ln k <$$

-0.00055

/e find for the lower border of the three years WMAP data band:

$$\text{trinomial potential new inflation for } n_s = 0.93 : \quad 0.003 < r < 0.015 \quad \text{and} \quad -0.0011 < dn_s/d\ln k <$$

-0.00033

loreover, in new inflation with the trinomial potential, we find that n_s is bounded from above by

$$\text{new inflation :} \qquad n_s < n_{s\ maximum} = 0.961528\ldots \,.$$

or $n_s = 0.961528\ldots$ we have in this model $r = 0.114769\ldots$ (see figs. 4 and 6). Interestingly enoug vo values (two branches) of r for one value of n_s in the interval $0.96 < n_s < 0.961528\ldots$ [see fig. $_{max} = 0.114769\ldots$ is the maximun r in the first branch. The values $0.16 \geq r \geq 0.114769\ldots$ correspo

(12)

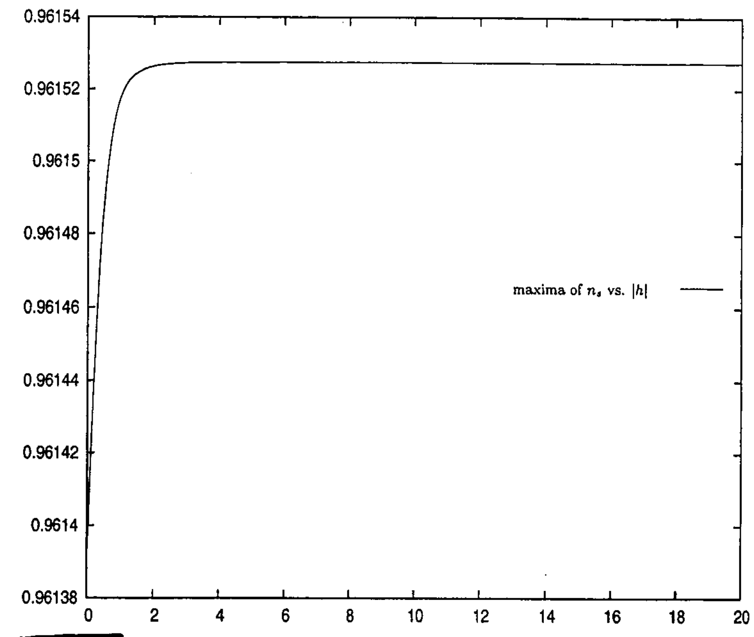

FIG. 6: $\boxed{\text{New Inflation.}}$ Maxima of n_s plotted vs. the asymmetry of the potential $|h|$. The limiting value for large $|h|$ is $n_s^{maximum} = 0.961528\ldots$

- n_s has an absolute maximum value

  $n_s = 0.961528\ldots\ldots$

- No lower bound for n_s

If n_s is observed larger than $0.962\ldots$, New Inflation Trinomial Potential should be excluded

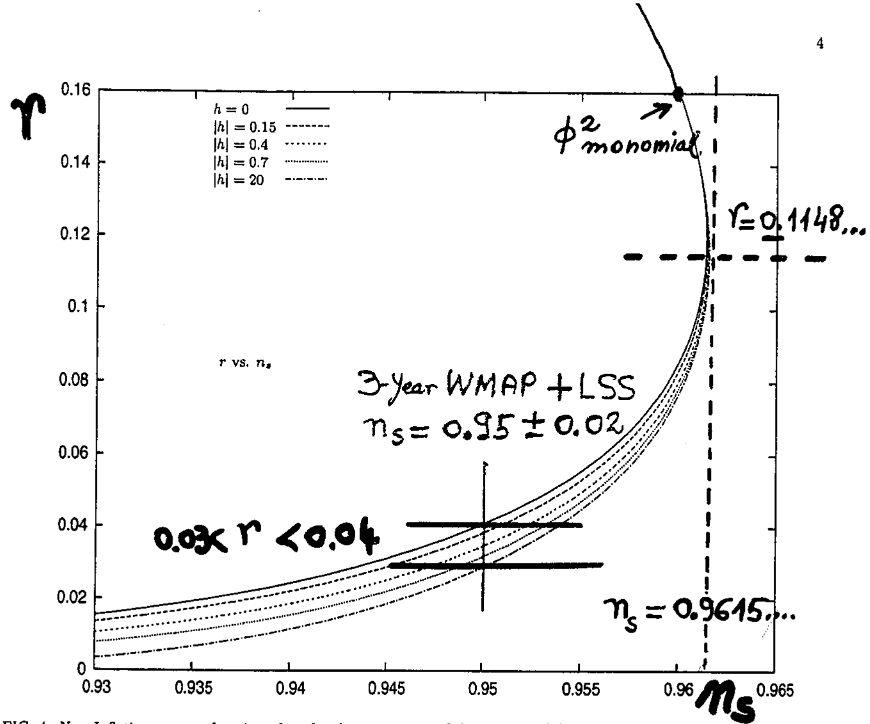

FIG. 4: New Inflation. r as a function of n_s for the asymmetry of the potential $|h| = 0, 0.15, 0.4, 0.7$ and 20. For a given n_s, r monotonically and slowly decreases with increasing $|h|$. $r = r(n_s)$ is not too sensitive to h. The maximun value of n_s is $n_s^{maximum} = 0.961528\ldots$ and the corresponding r is $r_{max} = 0.114769\ldots$. The maximun value of r is $r_{abs\ max} = 0.16$ and corresponds to the quadratic potential setting $y = 0$ in eq.(3.2). For $n_s = 0.95$ (the three years WMAP value), we find $0.03 < r < 0.04$.

PREDICTIONS for r :

NEW INFLATION

(r vs n_s)

Running spectral index $\frac{dn_s}{d\ln k}$

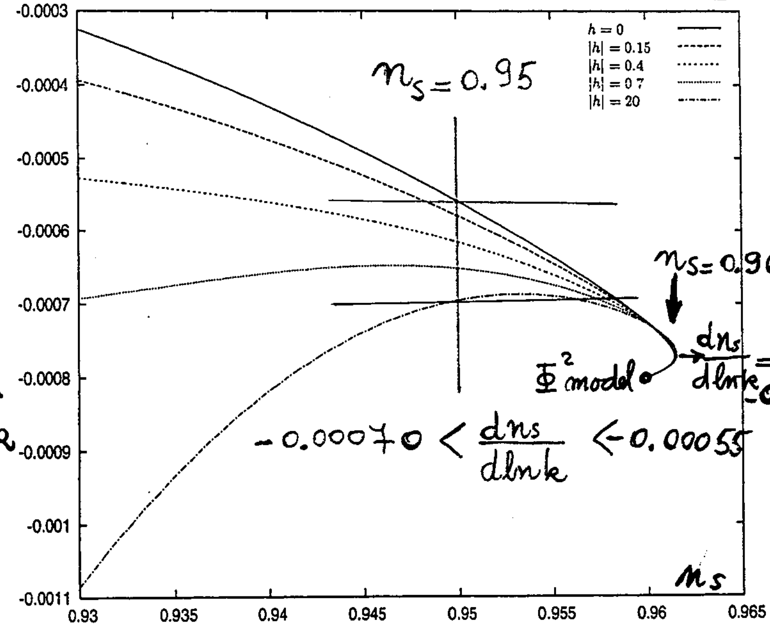

FIG. 5: New Inflation. The running $dn_s/d\ln k$ as a function of n_s for the asymmetry of the potential $|h| = 0, 0.15, 0.4, 0.7$ and 20. The running turns out to be always negative in new inflation. For $n_s < 0.96$, the running $dn_s/d\ln k$ decreases with increasing $|h|$. The opposite happens for $n_s > 0.96$. In the last case the dependence on h is weak. We find $dn_s/d\ln k = -0.00077\ldots$ at the branch point $n_s = 0.961\ldots$ for all $|h|$. The point $n_s = 1 - \frac{2}{N} = 0.96$, $\frac{dn_s}{d\ln k} = -\frac{2}{N^2} = -0.0008$ is reached for all values of h and corresponds to the monomial potential eq.(5.2). For $n_s = 0.95$ (the three years WMAP value), we find $-0.00070 < dn_s/d\ln k < -0.00055$.

$\frac{dn_s}{d\ln k}$ is a two valued function of n_s

NEW INFLATION

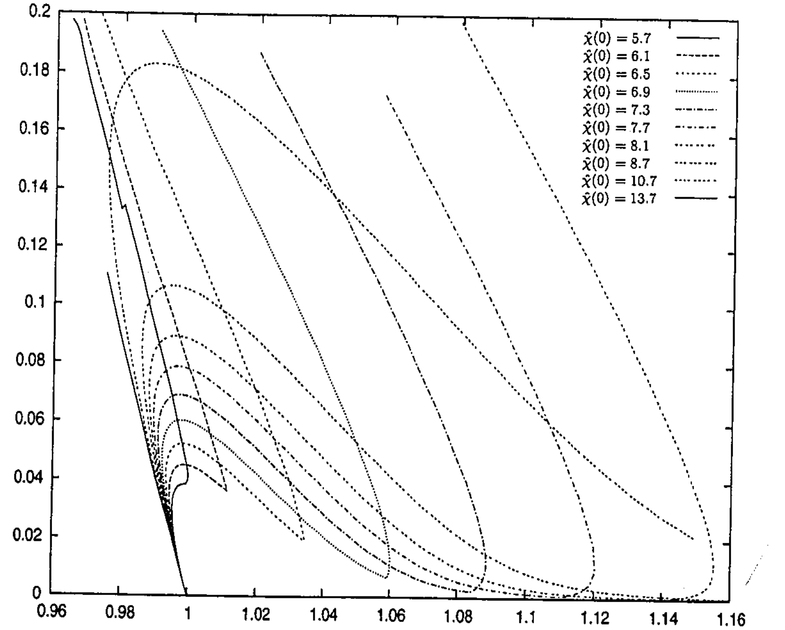

FIG. 21: Hybrid inflation. The ratio r vs. n_s for $\mu^2 = 1.7\,\Lambda > \mu^2_{crit}$, $g^2 = \frac{1}{4}$ and $\chi(0)_{crit} \simeq 5.7\sqrt{\Lambda} \leq \chi(0) \leq 13.7\sqrt{\Lambda}$. Notice that we have here both $n_s > 1$ and $n_s < 1$ depending on the values of Λ and $\tilde{\chi}(0)$. All curves end $[\tilde{\chi}(0) < \tilde{\chi}(0)_{crit}]$ or start $[\tilde{\chi}(0) > \tilde{\chi}(0)_{crit}]$ at $n_s = 1$ which is the fixed point for all values of $\tilde{\chi}(0)$.

HYBRID INFLATION PREDICTIONS

r vs n_s

BOTH $n_s < 1$ and $n_s > 1$ regimes

$n_s = 1$: fixed Point for ALL $\chi(0)$

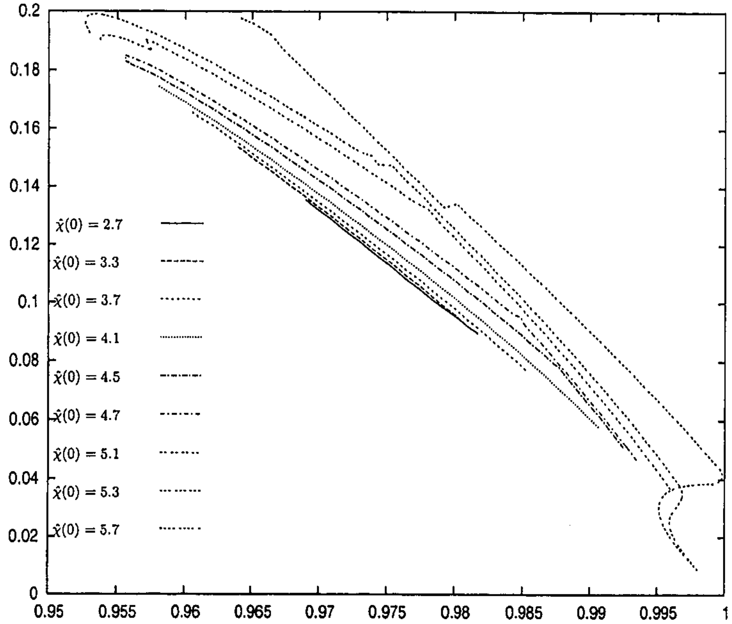

FIG. 20: Hybrid inflation. The ratio r vs. n_s for $\mu^2 = 1.7\,\Lambda > \mu^2_{crit}$, $g^2 = \frac{1}{4}$ and $2.7\,\sqrt{\Lambda} \le \chi(0) \le 5.7\,\sqrt{\Lambda} \simeq \chi(0)_{crit}$. Notice that here $n_s < 1$ for all values of Λ and this range of $\hat\chi(0)$. We see that $0.2 > r > 0.14$ for the interval $0.952 < n_s < 0.97$.

HYBRID INFLATION PREDICTIONS for r vs n_s

(RED TILTED REGIME)
$0.952 < n_s < 0.97$
$0.2 > r > 0.14$

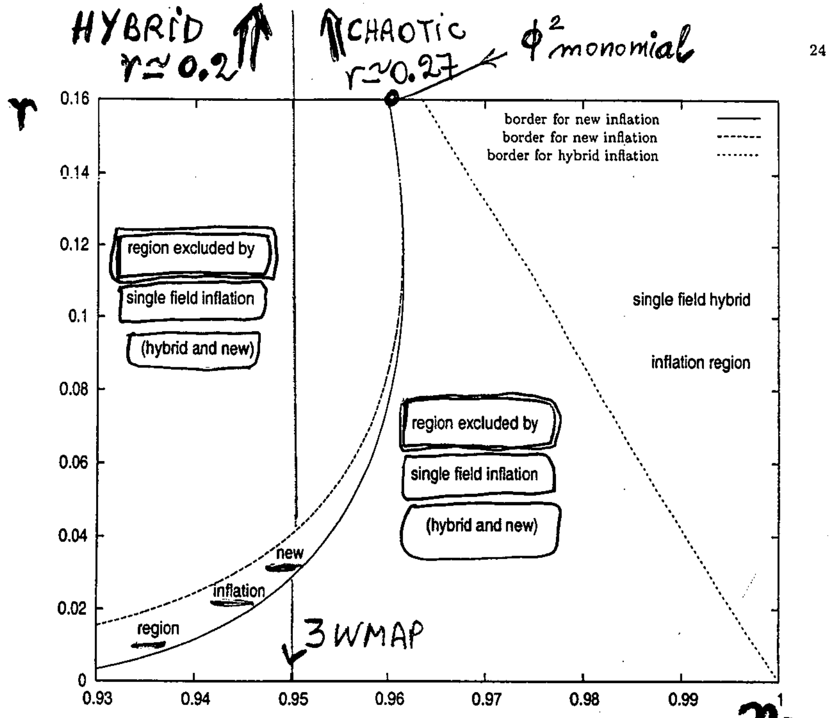

FIG. 24: Regions described in the (r, n_s)-plane by new and single field hybrid inflation for $n_s < 1$. The hybrid inflation border corresponds to $\mu^2 = 1.7\,\Lambda$. For $n_s > 1$, all values of (r, n_s) can be described by hybrid inflation (at least for $r < 0.2$, $n_s < 1.15$). The excluded region cannot be described by single field inflation (neither hybrid inflation, nor new inflation). Two or more fields inflation could describe such regions.

PREDICTIONS : theoretically allowed and discarded regions in the (r, n_s) plane for :

NEW INFLATION and

HYBRID INFLATION

We find a simple relation between the Inflation Energy scale M, n_s and r :

$$\boxed{10^4 \left(\frac{M}{M_{Pl}}\right)^2 = 1.27 \sqrt{r(1-n_s)}}$$

SMALL AND LARGE FIELD INFLATION

$$\boxed{10^4 \left(\frac{M}{M_{Pl}}\right)^2 = 1.27 \sqrt{r\left(n_s - 1 + \frac{3}{8} r\right)}}$$

HYBRID INFLATION

$$1.27 = 5\pi\sqrt{3}\ 10^3 \left|\delta^{(S)}_{k\,ad}\right|$$

$$\left|\delta^{(S)}_{k\,ad}\right| = 0.467\ 10^{-4}\ \text{WMAP}$$

WMAP 3 yrs.
A.G. Sánchez et al.
MNRAS, 366, 189 (2006)

$$\left.\begin{array}{l} n_s = 0.95 \pm 0.02 \\ \text{red tilted} \end{array}\right.$$

From this value, NEW INFLATION Trinomial potential **predicts**

$$0.03 < r < 0.04$$

$$-0.0007 < \frac{dn_s}{d\ln k} < -0.0055$$

Chaotic INFLATION Trinomial potential predicts $r = 0.27$

WMAP 3 yrs + SDSS says that $r < 0.28$ (95% CL)

$\Rightarrow$ This **DISFAVOURS** CHAOTIC INFLATION

Therefore, although the WMAP value for n_s [eq.(1.6)] is compatible both with chaotic an
WMAP bounds on r **clearly disfavour** chaotic inflation. New inflation easily fulfils the three y
on r and prepares the way for the expected data on the ratio of tensor/scalar fluctuations $r \lesssim 0.$

In the inflationary models of hybrid type, the inflaton is coupled to another scalar field σ_0 with
through a potential of the type [16]

$$V_{hyb}(\phi, \sigma_0) = \frac{m^2}{2}\,\phi^2 + \frac{g_0^2}{2}\,\phi^2\,\sigma_0^2 + \frac{\mu_0^4}{16\,\Lambda_0}\left(\sigma_0^2 - \frac{4\,\Lambda_0}{\mu_0^2}\right)^2 =$$
$$= \frac{m^2}{2}\,\phi^2 + \Lambda_0 + \frac{1}{2}\,(g_0^2\,\phi^2 - \mu_0^2)\,\sigma_0^2 + \frac{\mu_0^4}{16\,\Lambda_0}\,\sigma_0^4\,,$$

where $m^2 > 0$, $\Lambda_0 > 0$ plays the role of a cosmological constant and g_0^2 couples σ_0 with ϕ.

The initial conditions are chosen such that σ_0 and $\dot\sigma_0$ are very small (but not identically zero) a
is driven by the cosmological constant Λ_0 plus the initial value of the inflaton $\phi(0)$. The inflato
with time while the scale factor $a(t)$ grows exponentially with time. The field σ_0 has an effective

$$m_\sigma^2 = g_0^2\,\phi^2 - \mu_0^2\,.$$

Since the inflaton field ϕ decreases with time, m_σ^2 becomes negative at some moment durin
moment, spinodal (tachyonic) unstabilities appear and the field σ_0 starts to grow exponentially.
both fields ϕ and σ_0 are comparable with $\dot\phi$ and $\dot\sigma_0$ and close to their vaccum values.
We find that the time when the effective mass of the field σ_0 eq.(1.9) becomes negative dep

where the $\pm 6\%$ correspond to the error bars in the amplitude of adiabatic perturbations[8]. From figs. 9, 12 and 15 we can understand how the mass ratio $\frac{m}{M_{Pl}}$ varies with n_s and r. We find a **limiting** value $x_0 \equiv 10^5\,\frac{m_0}{M_{Pl}} \simeq 1$ for the inflaton mass such that $m_0 \simeq 10^{-5}\,M_{Pl}$ is a **minimal** inflaton mass for chaotic inflation, and a **maximal** mass for new inflation in order to keep n_s and r within the WMAP data.

New inflation arises for broken symmetric potentials (the minus sign in front of the φ^2 term) while chaotic inflation appears both for unbroken and broken symmetric potentials. For broken symmetry, we find that analytic continuation connects the observables for chaotic and new inflation: the observables are **two-valued** functions of $y \equiv \kappa N$. (N being the number of efolds from the first horizon crossing to the end of inflation). One branch corresponds to new inflation and the other branch to chaotic inflation. As shown in figs. 4-7, 9, 12 and 15, n_s, r and $|\delta^{(S)}_{k\;ad}|^2$ for chaotic inflation are connected by analytic continuation with the same quantities for new inflation. The branch point where the two scenarios connect corresponds to the monomial $+\varphi^2$ potential ($\kappa = \gamma = 0$).

The potential which **best fits** the present data for $n_s < 1$ and which **best prepares** the way to the expected data (a small $r \lesssim 0.1$) is given by the trinomial potential eq.(1.1) with a negative φ^2 term, that is **new** inflation.
In new inflation we have the upper bound

$$r \leq \frac{8}{N} \simeq 0.16 \quad .$$

NEW INFLATION

This upper bound is attained by the quadratic monomial. On the contrary, in chaotic inflation for both signs of the φ^2 term, r is bounded as

$$0.16 \simeq \frac{8}{N} < r < \frac{16}{N} \simeq 0.32 \quad ,$$

CHAOTIC INFLATION