Primordial non-Gaussianity in the CMB anisotropies

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Plan of the talk

- Introduction to primordial non-Gaussianity
- Second-order cosmological perturbations and ***CMB non-Gaussianity from Inflation and alternative scenarios***
- Fully non-linear CMB anisotropies on large-scales
- Imprints of primordial non-Gaussianity in the Large-Scale Structures

Based on**:**

- *Acquaviva V., B.N., Matarrese S., Riotto A., 2003, Nucl. Phys. B* **667**
- *B. N., Matarrese S., Riotto A. , 2004, JCAP* **01** *003*
- *B. N., Matarrese S., Riotto A., 2004 Phys. Rev.D* **69** *043503*
- *B. N., Matarrese S., Riotto A., 2004 JHEP* **04** *006*
- **B. N., Matarrese S., Riotto A., Komatsu E., 2004 Phys. Rept. 402 103**
- *B. N., Matarrese S., Riotto A., 2004, Phys. Rev. Lett.* **93** *331301*
- *B. N., Matarrese S., Riotto A., 2004, astro-ph/050641*
- *B. N., Matarrese S., Riotto A., 2004, astro-ph/0501614*

Scenarios to generate the primordial density perturbations

♦ **Standard scenario:** perturbations due to the fluctuations of the inflaton field itself leading to primordial adiabatic perturbations

♦ **Curvaton scenario** initial isocurvature perturbations associated to quantum fluctuations of a light scalar field different from the inflaton -- the curvaton σ -- with negligible energy density during inflation. The curvaton isocurvature perturbations are transformed into adiabatic when the curvaton decays into radiation after the end of inflation

K. Enqvist & M.S. Sloth, 2001
T. Moroi & T. Takahashi, 2001
D. Lyth & D. Wands, 2001

♦ **Modulated (Inhomogeneous) reheating**: fluctuations in the decay rate of the inflaton field $\delta\Gamma_\varphi$ due to some other light field. Adiabatic perturbations in the final reheating temperature in different regions of the universe.

G. Dvali, A. Gruzinov, M. Zaldarriaga, 2003
Kofman, 2003
(see also T. Hamazaki and H. Kodama, 1996)

….and some alternative models of inflation (multi-field inflation, Ghost inflation, D-cceleration….)

A basic parametrization

- A phenomenological way of parametrizing the possible presence of non-Gaussianity is the formula
(*e.g. Verde et al 2001; Komatsu & Spergel 2001)*

$$\Phi = \Phi_L + f_{NL} * \left(\Phi_L^2 - \langle \Phi_L^2 \rangle \right)$$

where Φ is the large-scale (Bardeen) gravitational potential, Φ_{L} its linear Gaussian part and f_{NL} is the so called **non-linearity parameter**

$$\delta g_{00} = -a^2(\tau)(1+2\Phi)$$

- This Non-Gaussian feature is then transferred to the **large-scale** CMB anisotropies through the Sachs-Wolfe effect

$$\frac{\Delta T}{\mathrm{T}} = \frac{1}{3}\Phi$$

What if the perturbations are (Non) Gaussian?

If the fluctuations $\delta(\mathbf{x})$ are Gaussian distributed then all the even n-point function $\langle \delta(\mathbf{x_1})\delta(\mathbf{x_2}) \,.....\, \delta(\mathbf{x_n}) \rangle$ are expressed in terms of the two-point correlation function (or the power spectrum) $\langle \delta(\mathbf{x_1})\delta(\mathbf{x_2}) \rangle$ while the odd ones vanish.

Thus a non-vanishing **three point function**, or its Fourier transform, the **bispectrum**

$$\langle \delta(\mathbf{x_1})\delta(\mathbf{x_2})\delta(\mathbf{x_3}) \rangle \neq 0$$

is an indicator of non-Gaussianity

e.g. : the $\Phi(\mathbf{k})$-bispectrum

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3) \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \times [2 f_{NL} P_\Phi(\mathrm{k}_1) P_\Phi(\mathrm{k}_2) + cycl]$$

where $\langle \Phi_L(\mathbf{k})\Phi_L(\mathbf{k}') \rangle = (2\pi)^3 P_\Phi(\mathrm{k}_1)\ \delta^{(3)}(\mathbf{k} + \mathbf{k}')$ is the linear power spectrum

Observational limits on f_{NL}

Tightest limits to date from WMAP

$$-58 < f_{\rm NL} < 134 \qquad (95\%)$$

by an analysis of the bispectrum
of the CMB temperature anisotropies
Komatsu et al. (2003)

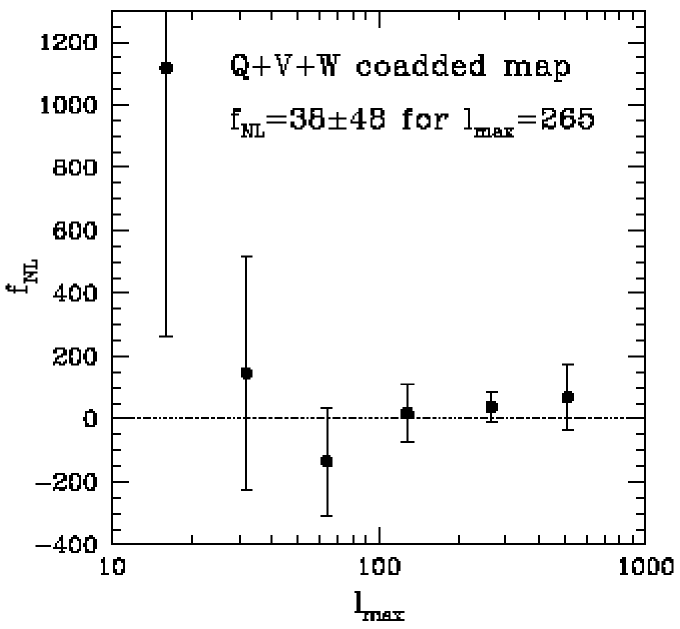

Fig. 1.— The non-linear coupling parameter $f_{\rm NL}$ as a function of the maximum multipole $l_{\rm max}$, measured from the Q+V+W coadded map using the cubic (bispectrum) estimator [Eq. (8)]. The best constraint is obtained from $l_{\rm max} = 265$. The distribution is cumulative, so that the error bars at each $l_{\rm max}$ are not independent.

The minimum value of $|f_{\rm NL}|$ detectable using the bispectrum analysis is 20 with WMAP and 5 with Planck data
Komatsu & Spergel (2001)

Origin of large-scale Non-Gaussianity

How do the perturbations that we observe (CMB anisotropies, Large-scale structures) contain information about the primordial level of non-Gaussianity?

Non-Gaussianity ⟷ **interactions**

self-interactions of the inflaton field φ
interactions with other scalar fields χ
non-linearities in the energy density $\propto \sigma^2$

↓

non-Gaussian fluctuations in the gravitational potential (gravitational non-linearties, mode-mode couplings)

↓

non-Gaussian CMB anisotropies

Primordial density perturbations are generated during (or after) inflation and then they evolve in the radiation/matter dominated epochs acquiring **different levels of non-linearities**

$$f_{NL} \equiv f_{NL}(\mathbf{k}_1, \mathbf{k}_2; \alpha, \beta, \mu..)$$

Scale-dependence

Parameters of the models

Why Non-Gaussianity is important

It can bring crucial information on the mechanism which generated the cosmological perturbations.

WMAP and *Planck* will allow to constrain or detect non-Gaussianity of CMB anisotropies with high precision

GOAL: Accurate calculation of the primordial bispectrum (and higher statistics) of the cosmological perturbations

Some hystory…..

- From the non-linearity in the inflaton potential in a de Sitter background *Falk et al. (1993)* found **f_{NL}~ O(ε^2)** in the standard single-field slow-roll inflation

- Using stochastic approach to inflation, *Gangui et al. (1994)* showed that indeed the main contribution come from the second-order perturbations in the metric yielding **f_{NL}~ O(ε)**

- This result has been obtained in a more regorous way by *Acquaviva, N.B., Matarrese, Riotto (2003)* and *Maldacena (2003)* with a full second-order calculation

- *N.B., Matarrese, Riotto (2004)* find that **after inflation** the non-Gaussian signal gets enhanced **f_{NL}~ 1** due to the non-linear gravitational dynamics

Linear perturbations

$$ds^2 = a^2(\tau)[-(1+2\phi)d\tau^2 - \omega_i \, d\tau \, dx^i + ((1-2\psi)\delta_{ij} + h_{ij})dx^i dx^j]$$

The energy conservation equation for linear density perturbations on *superhorizon* scales

$$\delta\rho' = -3H(\delta\rho + \delta P) - 3\psi'(\rho + P)$$

can be rewritten in terms of the ***curvature perturbation*** ζ

$$\zeta \equiv -\psi - H\frac{\delta\rho}{\rho'}$$

$$\zeta' = -\frac{H}{P+\rho}\delta P_{\mathrm{nad}}$$

$\delta P_{\mathrm{nad}} = \delta P - c_S^2 \, \delta\rho$ *non-adiabatic component* of the pressure perturbation, $c_S^2 = P'/\rho'$

D. Wands, K. Malik, D. Lyth and A. R. Liddle, 2000

First introduced by *Bardeeen, Steinhardt, Turner, 1989*

Primordial linear density perturbations

- **Standard scenario**

  ζ is generated from the ***inflaton fluctuations*** and remains constant on superhorizon scale during inflation/reheating and after inflation

$$\zeta \cong \zeta_{\mathbf{in}} = \left(\frac{H}{\dot{\varphi}} \delta\varphi \right)_*$$

- **Curvaton scenario**

  Initial ***isocurvature perturbation*** $\zeta_\sigma \approx 2(\delta\sigma/\sigma)_*$ in the ***curvaton field σ*** converted in adiabatic perturbations when the curvaton decays into radiation after inflation.

$$\zeta = r\zeta_\sigma, \quad r = (\rho_\sigma/\rho)_{\mathbf{decay}}$$

remains constant on large-scales afterwards (in the radiaion, matter,…. epochs)

- **Modulated Reheating**

  ζ is generated during reheating, from a ***fluctuating inflaton decay rate*** $\delta\Gamma_\varphi \propto \delta\chi$

$$\zeta = -\frac{1}{6}\frac{\delta\Gamma_\varphi}{\Gamma_\varphi}$$

remains constant afterwards (in the radiaion, matter,…. epochs)

Second-order cosmological perturbations

$$ds^2 = a^2(\tau)[-(1+2\phi)d\tau^2 - \omega_i\, d\tau\, dx^i + ((1-2\psi)\delta_{ij} + h_{ij})dx^i dx^j]$$

e.g. $\psi = \psi^{(1)} + \frac{1}{2}\psi^{(2)},$ $\qquad \delta\rho = \delta^{(1)}\rho + \frac{1}{2}\delta^{(2)}\rho$

Second-order gauge-invariant curvature perturbation (*Malik & Wands 2003)*

$$\zeta = \zeta^{(1)} + \tfrac{1}{2}\zeta^{(2)}$$

$$\begin{aligned}\zeta^{(2)} &= -\psi^{(2)} - \mathcal{H}\frac{\delta^{(2)}\rho}{\rho'} + 2\mathcal{H}\frac{\delta^{(1)}\rho'}{\rho'}\frac{\delta^{(1)}\rho}{\rho'} \\ &+ 2\frac{\delta^{(1)}\rho}{\rho'}\left(\psi^{(1)\prime} + 2\mathcal{H}\psi^{(1)}\right) - \left(\frac{\delta^{(1)}\rho}{\rho'}\right)^2\left(\mathcal{H}\frac{\rho''}{\rho'} - \mathcal{H}' - 2\mathcal{H}^2\right)\end{aligned}$$

See also Lyth & Wands, 2003; N.B., Matarrese, Riotto 2002; Rigopoulous & Shellard 2003

Such a quantity is related to the analougous ***non-perturbative quantity*** defined by *Salopek and Bond* (1990) which expanded at second order is $\zeta_{SB}^{(2)} = \zeta^{(2)} - 2\left(\zeta^{(1)}\right)^2$

For recent non-perturbative definitions of ζ, see Rigopoulos et al. (2003), Kolb et al. (2004), Lyth et al (2004), Langlois & Vernizzi (2005)

From the energy-continuity equation at second-order on superhorizon scales

$$\zeta^{(2)\prime} = -\frac{H}{\rho + P}\delta^{(2)}P_{\mathbf{nad}} - \frac{2}{\rho + P}\left[\delta^{(1)}P_{\mathbf{nad}} - 2(\rho + P)\zeta^{(1)}\right]\zeta^{(1)\prime}$$

and

$$\zeta^{(1)\prime} = -\frac{H}{\rho + P}\delta^{(1)}P_{\mathbf{nad}}, \qquad \delta^{(1)}P_{\mathbf{nad}} = \delta^{(1)}P - c_S^2\,\delta^{(1)}\rho$$

The key point is that $\zeta^{(2)}$ remains ***constant*** on superhorizon scales after it has been generated and possible isocurvature (entropy) perturbations are no longer present.

Thus $\zeta^{(2)}$ provides all the information about the primodial level of the NG generated during inflation, as in the standard scenario, or after inflation, as in the curvaton scenario.

Theoretical determination of f_{NL}

- Evaluate the non-Gaussianity generated ***during inflation*** (or immediately after as in the curvaton scenario): **primordial "input''**

The value of $\zeta^{(2)}$ is different for different scenarios $\zeta^{(2)} = 2a_{NL}\left(\zeta^{(1)}\right)^2$

- ***After inflation***, follow the evolution of the non-linearities on large scales by matching the conserved quantity $\zeta^{(2)}$ to the initial input, *plus* Einstein equations at second-order: **non-linearities in the gravitational potentials**

- The non-linearities in the gravitational potentials are then transferred to the observable ΔT/T fluctuations: **additional non-linearities are acquired**

Post-inflationary nonlinear gravitational dynamics is common to all scenarios

NB: This procedure defines a non-Gaussian contribution on large scales, to be taken as initial conditions. Evolving it inside the horizon requires a *radiation transfer function at second-order in the perturbations*

Einstein equations (in the Poisson gauge) → second-order evolution of the gravitational potentials (for a matter-dominated universe)

$$\phi^{(2)} = -\frac{1}{2}\delta^{(2)}\rho/\rho + 4\left(\psi^{(1)}\right)^2$$

(0-0) - component

$$\nabla^4(\psi^{(2)} - \phi^{(2)}) = -4\nabla^4\left(\psi^{(1)}\right)^2 + 10\partial_i\partial^j\left(\partial^i\psi^{(1)}\partial_j\psi^{(1)}\right) - \frac{10}{3}\nabla^2\left(\partial^i\psi^{(1)}\partial_i\psi^{(1)}\right)$$

from traceless part of (i-j) - component

CMB anisotropies at second-order

Gauge-invariant temperature fluctuations at second-order on **large scales.**

$$\frac{\Delta T}{T} = \phi_{\mathcal{E}}^{(1)} + \tau_{\mathcal{E}}^{(1)} + \frac{1}{2}\left(\phi_{\mathcal{E}}^{(2)} + \tau_{\mathcal{E}}^{(2)}\right) - \frac{1}{2}\left(\phi_{\mathcal{E}}^{(1)}\right)^2 + \phi_{\mathcal{E}}^{(1)}\tau_{\mathcal{E}}^{(1)}$$

+ integrated effects

$\tau_\varepsilon = \tau_\varepsilon^{(1)} + \tau_\varepsilon^{(2)}/2$ is the intrinsic fractional temperature fluctuation at emission $\tau_\varepsilon = \Delta T/T|_\varepsilon$

Pyne & Carroll (1996); Mollerach & Matarrese (1997)

Sachs-Wolfe effect at second-order

$$\frac{\Delta T}{T} = \frac{1}{3}\left[\phi_{\mathcal{E}}^{(1)} + \frac{1}{2}\left(\phi_{\mathcal{E}}^{(2)} - \frac{5}{3}\left(\phi_{\mathcal{E}}^{(1)}\right)^2\right)\right]$$

N.B., Matarrese, Riotto, JCAP 2004

From inflation and post-inflation

Additional second-order corrections

Sachs-Wolfe effect: a compact formula

$$\frac{\Delta^{(2)}T}{T} = -\frac{4}{9}\left(\phi^{(1)}\right)^2 + \frac{1}{3}g - \frac{1}{10}\left[\zeta^{(2)} - 2\left(\zeta^{(1)}\right)^2\right]$$

Post-inflation non-linear evolution of gravity

$$g = 3/2\left(\psi^{(1)}\right)^2 + \nabla^{-2}\left(\partial^i\psi^{(1)}\partial_i\psi^{(1)}\right) - 3\nabla^{-4}\partial^i\partial_j\left(\partial^i\psi^{(1)}\partial_j\psi^{(1)}\right)$$

Initial conditions set during or after inflation

$$\zeta^{(2)} = 2a_{NL}\left(\zeta^{(1)}\right)^2$$

standard scenario

$$a_{NL} = 1 - \frac{1}{4}\left(n_\zeta - 1\right)$$

curvaton scenario

$$a_{NL} = \frac{3}{4r} - \frac{r}{2}$$

N.B., Matarrese, Riotto, Phys. Rev. Lett. (2004)

Extracting the non-linearity parameter $f_{\rm NL}$

$$\frac{\Delta T}{T} = \frac{1}{3}\left(\phi_{(1)} + f_{\rm NL} * \phi_{(1)}^2\right)$$

$$f_{\rm NL}(\mathbf{k}_1,\mathbf{k}_2) = \frac{5}{3}(a_{NL} - 1) + \frac{4}{3} - g(\mathbf{k}_1,\mathbf{k}_2)$$

Connection between theory and observations

This is the proper quantity measurable by CMB experiments, via the phenomenological analysis by *Komatsu and Spergel* (2001)

$$g(\mathbf{k}_1,\mathbf{k}_2) = 4\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k^2} - 3\frac{(\mathbf{k}_1 \cdot \mathbf{k}_2)^2}{k^4} + \frac{3}{2}\frac{k_1^4 + k_2^4}{k^4}$$

$$k = |\,\mathbf{k}_1 + \mathbf{k}_2\,|$$

Non-Gaussianity in the standard scenario (I)

Non-Gaussianity generated **during inflation:**

Accounting for the inflaton self-interactions ***and metric fluctuations at second-order in the perturbations*** brings

$$\zeta^{(2)} = 2a_{NL}\left(\zeta^{(1)}\right)^2 \qquad 1 - a_{NL} = \frac{1}{4}(2\eta - 6\varepsilon) + f(\mathbf{k}_1, \mathbf{k}_2)$$

Acquaviva, N.B., Matarrese, Riotto (2003); Maldacena (2003)

Non-Gaussianity for single-field models of slow-roll inflation is tiny *during inflation:*

$$|n-1| = |2\eta - 6\varepsilon| << 1 \qquad \varepsilon \equiv \frac{\mathbf{1}}{\mathbf{16\pi G}}\left(\frac{V'}{V}\right)^2, \; \eta \equiv \frac{1}{8\pi G}\frac{V''}{V}$$

Non-Gaussianity in the standard scenario (II)

What about the post-inflationary evolution ?

- $\zeta^{(2)}$ is ***conserved*** during the reheating stage and during radiation/matter phases

➡ Use second-order evolution of the gravitational potentials (in Poisson gauge)

$$f_{\rm NL}(\mathbf{k}_1,\mathbf{k}_2) = -\frac{5}{12}(n-1) + \frac{4}{3} - \mathrm{g}(\mathbf{k}_1,\mathbf{k}_2)$$

$$\mathrm{g}(\mathbf{k}_1,\mathbf{k}_2) = 4\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{\mathrm{k}^2} - 3\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{\mathrm{k}^4} + \frac{3}{2}\frac{\mathrm{k}_1^4+\mathrm{k}_2^4}{\mathrm{k}^4}$$

$\mathrm{k} = |\,\mathbf{k}_1 + \mathbf{k}_2\,|$

$f_{\rm NL} \sim \mathrm{O}(1)$

N.B., Matarrese, Riotto (2003)

Thus the main contribution to the non-Gaussian signal comes from the non-linear gravitational dynamics in the post-inflationary stages

Non-Gaussianity in the curvaton scenario

- **Evolution** for $\zeta^{(2)}$ $\quad \zeta^{(2)} = r\zeta_\sigma^{(2)} + r(1-r^2)\left(\zeta_\sigma^{(1)}\right)^2 \qquad r \simeq \left(\frac{\rho_\sigma}{\rho}\right)_{\rm DECAY}$

Set a different matching condition

$$\zeta^{(2)} = 2a_{NL}\left(\zeta^{(1)}\right)^2, \qquad a_{NL} = \frac{3}{4r} - \frac{r}{2}$$

- During radiation/matter phases $\zeta^{(2)}$ is *conserved*
- Use second-order evolution of the gravitational potentials

Curvaton scenario prediction

$$f_{\rm NL}(\mathbf{k}_1,\mathbf{k}_2) = \left[-\frac{1}{3} - \frac{5}{6}r + \frac{5}{4r}\right] - g(\mathbf{k}_1,\mathbf{k}_2)$$

N.B., Matarrese, Riotto (2003)

$f_{\rm NL} >> 1$ if $r << 1$

Lyht, Ungarelli, Wands (2002)

Non-Gaussianity in the mod. reheating scenario

- **Evolution** for $\zeta^{(2)}$: $\zeta^{(2)\prime} = -\frac{1}{3}\delta^{(2)}\Gamma_\varphi - \zeta^{(1)}\delta^{(1)}\Gamma_\varphi - 2\zeta^{(1)\prime}\zeta^{(1)} - \frac{2}{3}\left(\zeta^{(1)}\delta^{(1)}\Gamma_\varphi / H\right)$

Set a different matching condition

$$\zeta^{(2)} = 2a_{NL}\left(\zeta^{(1)}\right)^2, \qquad a_{NL} = -\frac{3}{5}I + \frac{1}{4}$$

- During radiation/matter phases $\zeta^{(2)}$ is *conserved*
- Use second-order evolution of the gravitational potentials

Inhom. reheating prediction

$$f_{\rm NL}(\mathbf{k}_1,\mathbf{k}_2) = \frac{1}{12} - I - g(\mathbf{k}_1,\mathbf{k}_2)$$

"Minimal case" $I = -5/2 + (5/12)(\Gamma/\alpha\Gamma_1) = 0$

N.B., Matarrese, Riotto (2003)

Alternative models of inflation

- **Multiple-field inflation:**
  Generically giving rise to cross-correlated and non-Gaussian isocurvature and adiabatic perturbations

N.B, Matarrese, Riotto 2002; Bernardeau & Uzan 2002; Bernardeau et al 2004; Enqvist & Vaihkonen 2004; Rigopoulous & Shellard 2004

- **Unconventional inflation set-ups**

  - Ghost inflation (*Arkani-Hamed et al. 2004*)

  - D-cceleration (*Alishahiha et al. 2004; Silverstein & Tong 2004*)

In all these models it is possible to reach $f_{NL} \sim 100$ in the adiabatic perturbation mode

Also: features in the inflaton potential (*Wang & Kamionkowsky 2000);* preheating models (*Enqvist et al*. 2004, 2005); de Vega's and Shellard's talks

Predicted values of f_{NL}

	$f_{\rm NL}(\mathbf{k}_1, \mathbf{k}_2)$	Comments
Single–field inflation	$\frac{4}{3} - g(\mathbf{k}_1, \mathbf{k}_2)$	$g(\mathbf{k}_1, \mathbf{k}_2) = 4\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k^2} - 3\frac{(\mathbf{k}_1 \cdot \mathbf{k}_2)^2}{k^4} + \frac{3}{2}\frac{k_1^4 + k_2^4}{k^4}$
Curvaton scenario	$\left[-\frac{1}{3} - \frac{5}{6}r + \frac{5}{4r}\right] - g(\mathbf{k}_1, \mathbf{k}_2)$	$r \approx \left(\frac{\rho_\sigma}{\rho}\right)_{decay}$
Inhomogeneous reheating	$\frac{1}{12} - I - g(\mathbf{k}_1, \mathbf{k}_2)$	$I = -\frac{5}{2} + \frac{5}{12}\frac{\bar{\Gamma}}{\alpha \Gamma_1}$ "minimal case" $I = 0$ $(\alpha = \frac{1}{6}, \Gamma_1 = \bar{\Gamma})$
Multiple scalar fields	Up to 100	order of magnitude estimate of the absolute value
"Unconventional" inflation set-ups		
Warm inflation	$-\frac{5}{6}\left(\frac{\dot{\varphi}_0}{H^2}\right)\left[\ln\left(\frac{\Gamma}{H}\right)\frac{V'''}{\Gamma}\right]$ $\sim 10^{-1}$	second–order corrections not included
Ghost inflation	$-85 \cdot \beta \cdot \alpha^{-8/5}$	post–inflationary corrections not included
D–cceleration	$-0.06\,\gamma^2$	post–inflationary corrections not included

In the region **f_{NL} ~ 1** second-order corrections are significant. Important to control the different non-linearities generated in different stages of the perturbation evolution to fully exploit Planck data

Fully non-linear CMB anisotropies on large scales

B.N., Matarrese S., Riotto A., astro-ph/0506410

Linear scalar perturbations $ds^2 = -(1+2\Phi_1)\,dt^2 + a^2(t)(1-2\Psi_1)\,\delta_{ij}\,dx^i dx^j$

Non-linear generalization $ds^2 = -e^{2\Phi}\,dt^2 + a^2(t)\,e^{2\Psi}\,\delta_{ij}\,dx^i dx^j$

Salopek-Bond (1990) likewise definition of the non-linear ζ quantity

$$g_{\mathbf{ij}} \approx a^2(t)\,e^{2\zeta}\,h_{\mathbf{ij}}\,dx^i dx^j$$

Split the gravitational potential $\Phi = \Phi_{\mathbf{long}} + \Phi_{\mathbf{short}}$ with $\Phi_{\mathbf{long}}$ a collection of modes with ***wavelengths larger than the Hubble radius*** (at last scattering)

A local observer will see the long-wavelength part as a homogeneous background

$$d\bar{t} = e^{\Phi_l}\,dt, \qquad \bar{a} = a(t)\,e^{\Psi_l}$$

$$ds^2 = -d\bar{t}^2 + \bar{a}^2\,\delta_{ij}\,dx^i dx^j$$

Non-Linear Sachs Wolfe effect (I)

For the large-scale CMB anisotropies the effects of the super-Hubble modes are encoded in the "barred metric" and one can adopt a locally homogeneous Universe

CMB temperature measured by O $\bar{T}_O = \frac{\bar{\omega}_O}{\bar{\omega}_\varepsilon} \bar{T}_\varepsilon$

(I) Redshift from the last scattering surface ε to the observer O

(II) Intrinsic temperature at emission

(I) $$\frac{d\bar{t}_\varepsilon}{d\bar{t}_O} = \frac{\bar{\omega}_O}{\bar{\omega}_\varepsilon} = \frac{\omega_O}{\omega_\varepsilon} \, e^{\Phi_{l\varepsilon} - \Phi_{lO}}$$

(II) Adiabaticity condition $\bar{T}_\varepsilon \propto \bar{\rho}_\gamma^{1/4} \propto \bar{\rho}_{\mathbf{m}}^{1/3}$

From 0-0 Einstein equation $\bar{\rho}_{\mathbf{m}} = \rho_{\mathbf{m}} \, e^{-2\Phi_{\mathbf{l}}}$

$\Rightarrow$ $$\bar{T}_\varepsilon = T_\varepsilon \, e^{-2\Phi_l/3}$$

$$\bar{H} \equiv \frac{d \ln \bar{a}}{d\bar{t}}, \quad \bar{H}^2 = \frac{8\pi G_N}{3} \bar{\rho}_{\mathbf{m}}$$

Non-Linear Sachs Wolfe effect (II)

From (I)+(II) $\overline{T}_O = \frac{\omega_O}{\omega_\varepsilon} T_\varepsilon e^{\Phi_l/3}$

Large-scale CMB anisotropies from the effects of the super-horizon modes encoded in Φ_l

$$\frac{\delta_{\mathbf{np}} \overline{T}}{T} = e^{\Phi_l/3} - 1$$

Non-perturbative extension of the Sachs-Wolfe effect

N.B., S. Matarrese, A. Riotto astro-ph/0506410

Linear order $\frac{\delta^{(1)}T}{T} = \frac{1}{3}\Phi^{(1)}$

Second order $\frac{\delta^{(2)}T}{T} = \frac{1}{3}\Phi^{(2)} + \frac{1}{9}\left(\Phi^{(1)}\right)^2$ see expression in *N.B. , Komatsu, Matarrese, Riotto (2004)*

CMB anisotropies and the curvature ζ (I)

Express the non-linear CMB anisotropies in terms of the ***non-linear curvature perturbation*** ζ

- From energy continuity equation $\zeta = -\Psi + \frac{1}{3}\ln\frac{\bar{\rho}}{\rho} = -\Psi - \frac{2}{3}\Phi$

  is conserved on large scales at any-order so its value is fixed at the end of inflation. It encodes the initial conditions from Inflation.

- **Fully non-linear relation between the gravitational potentials Φ and Ψ** from the (i-j)-Einstein equations

$$\nabla^4(\Psi - \Phi) = -\frac{3}{2}\partial_i\partial^k(\Psi^{,i}\Psi_{,k}) + \frac{1}{2}\nabla^2(\Psi^{,i}\Psi_{,i}) + \frac{7}{2}\partial_i\partial^k(\Phi^{,i}\Phi_{,k}) + \frac{1}{2}\nabla^2(\Phi^{,i}\Phi_{,i}) + 3\partial_i\partial^k(\Phi^{,i}\Psi_{,k}) - \nabla^2(\Phi^{,i}\Psi_{,i})$$

or $\Psi = \Phi + K[\Phi, \Psi]$

Apply an **iterative procedure to express K as powers of ζ**

Accounting for the kernel

$$\zeta = -\Psi - \frac{2}{3}\Phi \equiv \zeta_L + (a_{\mathrm{NL}} - 1)\zeta_L^2 + (b_{NL} - 1)\zeta_L^3$$

$$\nabla^4(\Psi - \Phi) = -\frac{3}{2}\partial_i\partial^k(\Psi^{,i}\Psi_{,k}) + \frac{1}{2}\nabla^2(\Psi^{,i}\Psi_{,i}) + \frac{7}{2}\partial_i\partial^k(\Phi^{,i}\Phi_{,k}) + \frac{1}{2}\nabla^2(\Phi^{,i}\Phi_{,i})$$
$$+ 3\partial_i\partial^k(\Phi^{,i}\Psi_{,k}) - \nabla^2(\Phi^{,i}\Psi_{,i})$$

Iterative procedure

$$\Phi^{(0)} = -\frac{3}{5}\zeta_L$$

↓

$$\Phi^{(1)} = \Phi^{(0)} - \frac{3}{5}(a_{NL} - 1)(\zeta_L)^2 - \overline{K}\left[(\zeta_L)^2\right]$$

↓

$$\Phi^{(2)} = \Phi^{(1)} - \frac{3}{5}(b_{NL} - 1)(\zeta_L)^2 - 2\left(\frac{3}{5}\right)^3(a_{NL} - 1)\,\overline{K}\left[\zeta_L, (\zeta_L)^2\right] - 2\left(\frac{3}{5}\right)^5\overline{K}\left[\zeta_L, \overline{K}\left[(\zeta_L)^2\right]\right]$$

$$\overline{K}[(\cdot),(\cdot)] \equiv 5\nabla^{-4}\partial_i\partial^k[(\cdot)^{,i}(\cdot)_{,k}] - \frac{5}{3}\nabla^{-2}[(\cdot)^{,i}(\cdot)_{,i}]$$

$$\frac{\delta_{\mathbf{np}}\overline{T}}{T} = e^{\Phi_l/3} - 1$$

CMB anisotropies and the curvature ζ (II)

$$\frac{\delta_{\mathbf{np}} \overline{T}_O}{T_O} = e^{-\zeta_L/5 - K[\zeta_L]/5} - 1$$

Single-field models of inflation: $\zeta = \zeta_L$ is a Gaussian perturbation but in general the kernel $K \neq 0$, containing quadratic and higher order terms in the Gaussian variable ζ (non-Gaussianity from non-linear evolution of gravity after inflation)

For other scenarios: in addition ζ will contain an intrinsic primordial Non-Gaussianity

$$\zeta = \zeta_L + (a_{NL} - 1)\zeta_L^2 + (b_{NL} - 1)\zeta_L^3 + \ldots.$$

for single field-inflation a_{NL}=1, b_{NL}=1 (up to tiny deviations proportional to powers of the slow-roll parameters)

N-point connected correlation functions

$$W^{(n)}(\mathbf{x}_1,\ldots,\mathbf{x}_n) \equiv \left\langle e^{-\zeta_L(\mathbf{x_1})/5-K[\zeta_L](\mathbf{x_1})} \ldots . e^{-\zeta_L(\mathbf{x_n})/5-K[\zeta_L](\mathbf{x_n})} \right\rangle_{\mathbf{conn.}}$$

A useful example: $\left\langle e^{\varphi(\mathbf{x_1})} \ldots\ldots e^{\varphi(\mathbf{x_n})} \right\rangle = e^{\frac{1}{2}\sum_{i,j}\langle \varphi(\mathbf{x_i})\varphi(\mathbf{x_j})\rangle}$ where φ(x) is a Gaussian field

But we cannot use the "Gaussian approximation" $e^{-\zeta_L/5}$ since the kernel $K \neq 0$ for single-field models of inflation (unless for specific configurations K vanishes)

Accounting for the kernel K

Iterative procedure to express K in terms of powers of ζ K[ζ]

Borrow standard techniques used in quantum field theory to evaluate the n-point connected corr. functions for a self-interacting scalar field:

$$W^{(n)}(\mathbf{x}_1,\ldots,\mathbf{x}_n) = i^{-n} \left. \frac{\delta^n W[J]}{\delta J(\mathbf{x}_1)\ldots\delta J(\mathbf{x}_n)} \right|_{(J=0)}$$

perturb around the case K=0

$$W[J] = \ln Z[J], \quad Z[J] = \int D[\zeta] P[\zeta] e^{i\int d\mathbf{x}\, J(\mathbf{x})(e^{-\zeta/5-K(\zeta)}-1)}$$

Results: bispectrum (I)

Kernel K up to second order $K(\zeta)=\int d\mathbf{x}_1 d\mathbf{x}_2 K_2(\mathbf{x}-\mathbf{x}_1,\mathbf{x}-\mathbf{x}_2)\zeta_L(\mathbf{x}_1)\zeta_L(\mathbf{x}_2)$

K_2 inverse Fourier transform of

$$\tilde{K}_2(\mathbf{k}_1,\mathbf{k}_2)=(a_{NL}-1)+\frac{9}{5}\left[\frac{(\mathbf{k}_1\cdot(\mathbf{k}_1+\mathbf{k}_2))(\mathbf{k}_2\cdot(\mathbf{k}_1+\mathbf{k}_2))}{|\mathbf{k}_1+\mathbf{k}_2|^4}-\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)}{3|\mathbf{k}_1+\mathbf{k}_2|^2}\right]$$

$$W^{(3)}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)=\left\langle(e^{-\zeta_L(\mathbf{x}_1)/5}-1)(e^{-\zeta_L(\mathbf{x}_1)/5}-1)(e^{-\zeta_L(\mathbf{x}_1)/5}-1)\right\rangle_{\mathbf{conn.}}$$
$$-10\int d\mathbf{y}_1 d\mathbf{y}_2 \underline{K_2(\mathbf{x}_1-\mathbf{y}_1,\mathbf{x}_2-\mathbf{y}_2)}\left[w_2(\mathbf{x}_2,\mathbf{y}_1)w_2(\mathbf{x}_3,\mathbf{y}_2)+w_4(\mathbf{x}_2,\mathbf{x}_3,\mathbf{y}_1,\mathbf{y}_2)/2\right]+cycl.$$

$$w_2(\mathbf{x}_1,\mathbf{x}_2)\equiv-\frac{1}{5}\left\langle(e^{-\zeta_L(\mathbf{x}_1)/5}-1)\zeta_L(\mathbf{x}_2)\right\rangle_{\mathbf{conn.}}=-\frac{1}{25}e^{\langle\zeta^2\rangle}\left\langle\zeta_L(\mathbf{x}_1)\zeta(\mathbf{x}_2)\right\rangle$$

$$w_4(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4)\equiv\left\langle(e^{-\zeta_L(\mathbf{x}_1)/5}-1)(e^{-\zeta_L(\mathbf{x}_2)/5}-1)\zeta_L(\mathbf{x}_3)\zeta_L(\mathbf{x}_4)\right\rangle_{\mathbf{conn.}}$$

Results: bispectrum in the squeezed limit (II)

- For ***single-field inflation*** ($a_{NL}=1$)
- and in the ***"squeezed'' limit*** ($k_1 << k_2, k_3$) the kernel $K \to 0$

Exact non perturbative 3-point correlation function

$$\left\langle (e^{-\zeta_L(\mathbf{x}_1)/5}-1)(e^{-\zeta_L(\mathbf{x}_1)/5}-1)(e^{-\zeta_L(\mathbf{x}_1)/5}-1)\right\rangle_{\mathbf{conn.}} = W_0^{(2)}(\mathbf{x}_1,\mathbf{x}_2)W_0^{(2)}(\mathbf{x}_1,\mathbf{x}_3)+cyclic$$

$$+W_0^{(2)}(\mathbf{x}_1,\mathbf{x}_2)W_0^{(2)}(\mathbf{x}_2\mathbf{x}_3)W_0^{(2)}(\mathbf{x}_3,\mathbf{x}_1)$$

$$W_0^{(2)}(\mathbf{x}_i,\mathbf{x}_j)=e^{\langle\zeta^2\rangle/50}(e^{\langle\zeta_1\zeta_2\rangle/50}-1)\equiv\int\frac{d\mathbf{k}}{(2\pi)^3}e^{i\mathbf{k}\cdot(\mathbf{x}_i-\mathbf{x}_j)}P(k)$$

Bispectrum: $$\left\langle \frac{\delta_{\mathbf{np}}T(\mathbf{k}_1)}{T}\frac{\delta_{\mathbf{np}}T(\mathbf{k}_2)}{T}\frac{\delta_{\mathbf{np}}T(\mathbf{k}_3)}{T}\right\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

$$B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=2P(k_1)P(k_2)+\int\frac{d\mathbf{q}}{(2\pi)^3}P(|\mathbf{q}|)P(|\mathbf{q}-\mathbf{k}_2|)P(|\mathbf{q}+\mathbf{k}_2|)\quad (k_1<<k_2,k_3)$$

For ***alternative scenarios*** and ***generic momenta configuration we recover the expression for the non-linearity parameter f_{NL}***

Results: trispectrum

$$\frac{\delta_{\mathbf{np}} \overline{T}_O}{T_O} = e^{-\zeta_L/5 - K[\zeta_L]/5} - 1 \qquad \zeta = \zeta_L + (a_{NL} - 1)\zeta_L^2 + (b_{NL} - 1)\zeta_L^3$$

Kernel K[ζ] expanded up to third order

For ***single-field inflation*** (a_{NL}=1, b_{NL}=1) and in the ***"squeezed" limit*** (k_1, k_2<< k_3, k_4) the kernel K→ 0

Exact non perturbative 4-point correlation function

$$\left\langle (e^{-\zeta_L(\mathbf{x}_1)/5} - 1)(e^{-\zeta_L(\mathbf{x}_2)/5} - 1)(e^{-\zeta_L(\mathbf{x}_3)/5} - 1)(e^{-\zeta_L(\mathbf{x}_4)/5} - 1) \right\rangle_{\mathbf{conn.}}$$

which is of the form $\left(W_0^{(2)}\right)^3 + \left(W_0^{(2)}\right)^4 + \left(W_0^{(2)}\right)^5 + \left(W_0^{(2)}\right)^6$

$$\left(W_0^{(2)}\right)^3 \sim W_0^{(2)}(\mathbf{x}_1, \mathbf{x}_2) W_0^{(2)}(\mathbf{x}_1, \mathbf{x}_3) W_0^{(2)}(\mathbf{x}_1, \mathbf{x}_4) + \ldots$$

Results: trispectrum (II)

For ***alternative scenarios*** and ***generic momenta configuration we can determine the non-linearity parameter g_{NL} which enters into the trispectrum of the CMB anisotropies, by expanding $e^{-\zeta/5-K[\zeta]}-1$***

$$\frac{\Delta T}{T}=\frac{1}{3}\Phi \qquad \Phi=\Phi_L+f_{NL}*\left(\Phi_L\right)^2+g_{NL}*\left(\Phi_L\right)^3$$

(e.g., Okamoto & Hu, 2002)

$$g_{NL}=\frac{25}{9}\left(b_{NL}-1\right)+\frac{5}{9}(a_{NL}-1)\left[5\mathbf{A}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)-1\right]+\frac{1}{54}+\frac{25}{9}\mathbf{C}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$
$$-\frac{1}{3}\left[\frac{(\mathbf{k}_1\cdot(\mathbf{k}_1+\mathbf{k}_2)(\mathbf{k}_2\cdot(\mathbf{k}_1+\mathbf{k}_2)}{|\mathbf{k}_1+\mathbf{k}_2|^2}-\frac{1}{3}\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{|\mathbf{k}_1+\mathbf{k}_2|^2}+cycl.\right]$$

$$\mathbf{A}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=\frac{6}{5}\left[\frac{\mathbf{k}_1\cdot\mathbf{k}_4(\mathbf{k}_3\cdot\mathbf{k}_4+\mathbf{k}_2\cdot\mathbf{k}_4)+(\mathbf{k}_2\cdot\mathbf{k}_4)(\mathbf{k}_3\cdot\mathbf{k}_4)}{k^4}-\frac{1}{3}\frac{\mathbf{k}_1\cdot(\mathbf{k}_2+\mathbf{k}_3)+\mathbf{k}_2\cdot\mathbf{k}_3}{k^2}\right]$$

$$\mathbf{C}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=\frac{54}{25}\frac{\mathbf{k}_4\cdot\mathbf{k}_3[(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{k}_4]}{k^2}\left[\frac{(\mathbf{k}_1\cdot(\mathbf{k}_1+\mathbf{k}_2)(\mathbf{k}_2\cdot(\mathbf{k}_1+\mathbf{k}_2)}{|\mathbf{k}_1+\mathbf{k}_2|^2}-\frac{1}{3}\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{|\mathbf{k}_1+\mathbf{k}_2|^2}\right]+cycl.$$

Imprints of primordial non-Gaussianity in the LSS

How the primordial NG set on ***superhorizon*** scales propagates to ***subhorizon*** scales in terms of dark matter density and velocity fields?

Solving Einstein equations at second-order ***with an initial NG set on superhorizon scales by a given scenario (e.g. inflation)***

$$\zeta^{(2)} = 2a_{NL}\left(\zeta^{(1)}\right)^2$$

$$\psi^{(2)} = -\frac{50}{9}a_{\mathbf{nl}}\varphi^2 + \frac{10}{27}\left(\frac{3}{4} - a_{NL}\right)\tau^2\varphi^{,k}\varphi_{,k} + \frac{10}{27}(1 - a_{NL})\tau^2\varphi\nabla^2\varphi - \frac{\tau^4}{252}\left(\frac{10}{3}\varphi^{,ik}\varphi_{,ik} - (\nabla^2\varphi)^2\right)$$

large-scale part ($\tau \to 0$)

The primordial NG propagates onto smaller scales when the mode reenters the horizon (terms $\propto\tau^2$)

Sincronous-gauge

$$ds^2 = a^2(\tau)[-d\tau^2 + ((1+2\psi)\delta_{ij} + \chi_{ij})dx^i dx^j]$$

$$\delta^{(1)} = \frac{\tau^2}{6}\nabla^2\varphi$$

$\varphi(\mathbf{x})$, *peculiar gravitational potential*

N.B., Matarrese, Riotto, astro-ph/0501614

Flowing of primordial NG onto sub-horizon scales

In the longitudinal (Poisson) gauge (***to confront with the Newtonian approach adopted in LSS observations***)

$$\begin{aligned}
\delta_P^{(2)} &= \frac{\tau^4}{126}\left[5\left(\nabla^2\varphi\right)^2 + 2\varphi^{,ij}\varphi_{,ij} + 7\varphi^{,i}\nabla^2\varphi_{,i}\right] + \frac{\tau^2}{9}\left[5a_{\rm nl}\nabla^2\varphi^2 + \frac{9}{2}\varphi_{,k}\varphi^{,k} + 16\varphi\nabla^2\varphi + \frac{36}{7}\Psi\right] \\
&+ \frac{20}{3}\left(a_{\rm nl} - \frac{2}{5}\right)\varphi^2 - 24\Theta \\
v_P^{(2)i} &= \frac{\tau^3}{9}\left(-\varphi^{,ij}\varphi_{,j} + \frac{6}{7}\Psi^{,i}\right) - 2\tau\left[\frac{10}{9}\left(\frac{21}{20} - a_{\rm nl}\right)\varphi\varphi^{,i} + 2\Theta^{,i}\right] + \frac{4\tau}{9}\varphi\varphi^{,i} - d^{i\prime}_{(2)}
\end{aligned}$$

$$\nabla^2\Theta \equiv \Psi - \varphi^{,i}\varphi_{,i}/3, \quad \Psi \equiv [\varphi^{,ik}\varphi_{,ik} - (\nabla^2\varphi)^2]/2$$

Newtonian terms

Post-Newtonian terms

The information on the primordial non-Gaussianity set on superhorizon scales flows into the Post-Newtonian terms

Primordial non-Gaussianity and Newtonian approach (I)

- Standard Newtonian approach

$$\frac{\tau^2}{6}\nabla^2\Phi = \delta$$

$$\Phi = \varphi + f_{\mathbf{nl}}\left(\varphi^2 - \langle\varphi^2\rangle\right) \rightarrow f_{\mathbf{nl}}\frac{\tau^2}{6}\nabla^2\varphi^2 = \delta^{(2)}$$

(e.g., *Verde et al. 2000)*

We find that primordial NG has a ***non trivial scale dependence***

$$f_{\rm nl}(\mathbf{k}_1,\mathbf{k}_2;\tau) = -\frac{5}{3}a_{\rm nl} + \frac{63(\mathbf{k}_1\cdot\mathbf{k}_2)+172(k_1^2+k_2^2)}{42k^2} + \frac{(k_1^2+k_2^2)(\mathbf{k}_1\cdot\mathbf{k}_2)}{k_1^2k_2^2k^2}\times\left(\frac{4}{7}(\mathbf{k}_1\cdot\mathbf{k}_2)+k_1^2+k_2^2\right) - \frac{6}{7}\frac{k_1^2k_2^2}{k^4} + \frac{6}{7}\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{k^4}$$

- Bispectrum of the matter density K($\mathbf{k}_1$, $\mathbf{k}_2$,τ)

$$\delta_{\mathbf{k}}(\tau) = \delta^{(1)}_{\mathbf{k}}(\tau) + \int\frac{d\mathbf{k}_1 d\mathbf{k}_2}{(2\pi)^3}\mathcal{K}(\mathbf{k}_1,\mathbf{k}_2;\tau)\delta^{(1)}_{\mathbf{k}_1}\delta^{(1)}_{\mathbf{k}_2}\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})$$

$$\mathcal{K}(\mathbf{k}_1,\mathbf{k}_2;\tau) = \frac{5}{7} + \frac{2}{7}\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{k_1^2k_2^2} + \frac{1}{2}\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)}{k_1^2k_2^2}\left(k_1^2+k_2^2\right) - 6f_{\rm nl}(\mathbf{k}_1,\mathbf{k}_2;\tau)\frac{k^2}{k_1^2k_2^2\tau^2}$$

Primordial non-Gaussianity and Newtonian approach (II)

Some remarks

- Primordial NG contributions ≅ Newtonian terms for $f_{NL} \approx \frac{10^5}{1+z}\left(\frac{Mpc}{\lambda}\right)^2$

At λ ~ few Mpc $f_{NL} \sim 10^3$ is required to have an observable imprints in the clusterings of galaxies at low redshifts
(more accurate analysis in *Verde, Wang, Heavens, Kamionkovski 2000; Verde, Jimenez, Kamionkowski, Matarrese 2004; Scocimarro, Sefusatti, Zaldarriaga 2004; Matarrese et al 1997)*

- Analogous results hold for the peculiar velocities. The standard Newtonian relation

$$\nabla \cdot v_P = -\frac{2}{3\mathcal{H}}\nabla^2\Phi$$

is recovered ($|a_{nl}| >> 1$)

- Easy to extend for Λ-CDM

Conclusions

- **For every scenario well defined predictions for the strength and shape of Non-Gaussianity.** In the standard scenario f_{NL} is (slightly) below the minimum value detectable by Planck f_{NL}~5 and the main contribution is from second-order metric perturbations, while alternative scenarios can produce stronger Non-Gaussianity than this minimum amount.

- **Prospects for the future:** investigation of **scale-dependence of f_{NL}** + CMB polarization+statistical tools complementary to the bispectrum+ simulated non-Gaussian maps can **push the sensitivity to non-Gaussianity down to the critical level of f_{NL}~1**

- Fully non-linear computation of large-scale CMB fluctuations allows to compute easily the non-linearity parameters f_{NL} and g_{NL}.

- **Constraining or detecting non-Gaussianity will be a powerful tool to discriminate among different scenarios for structure formation which would be indistinguishable with standard tests using the power-spectrum.**

“ As yet, all the tests carried out are consistent with initial perturbations, which are Gaussian, though the strength of this statement is unclear given the lack of so far well-motivated and calculable non-Gaussian models ”

A. Liddle & D. Lyth in “ Cosmological Inflation and Large-Scale Structure “