Effective field theory of inflation confronts CMB and LSS: a progress report

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- Effective field theory of inflation: condensate + fluctuations
- Small parameters: H/Mp + slow roll ⟶ expansion in 1/Ne
- Fluctuations: initial conditions, backreaction and the Quadrupole
- Model analysis and reconstruction program.

Effective field theory basics

A single scalar field

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi + V(\Phi)$$

$$\Phi(\vec{x}, t) = \phi + \delta\Phi(\vec{x}, t)$$

condensate

fluctuation

$$\delta\Psi \sim \frac{\delta T}{T}$$

$$g_{\mu\nu}(\vec{x}, t) = FRW + \delta g_{\mu\nu}(\vec{x}, t)$$

Tensor (grav.waves)

Vectors

Scalar

$$H^2 = \frac{8\pi}{3M_P^2}\left[\frac{\dot{\phi}^2}{2} + V(\phi)\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + V(\phi) = 0$$

$$\delta\ddot{\Psi}_k + 3H\delta\dot{\Psi}_k + \frac{k^2}{a^2(t)}\delta\Psi_k + \mathcal{W}(\phi)\delta\Psi_k = 0$$

- **Gravitational waves amplitude** $\sim \frac{H}{M_P} \ll 10^{-5}$
- **Nearly scale invariant spectrum of temperature fluctuations**
- **Need > 50 e-folds to solve horizon….problems**

$\Longrightarrow$ ***Small quantities*** $\rightarrow$ $\frac{H}{M_P}$ **Effective field theory**

$\rightarrow$ $dV(\phi)/d\phi, d^2V(\phi)/d\phi^2 \cdots \text{small}$ **Slow roll**

$$\epsilon_v = \frac{M_P^2}{2}\left[\frac{V'(\phi)}{V(\phi)}\right]^2, \eta_v = M_P^2 \frac{V''(\phi)}{V(\phi)}, \cdots$$

Slow roll parameters << 1

Curvature perturbations (Gaussian):

Boundary conditions and Quadrupole suppression

COBE, WMAPI, WMAPIII ⟶ ***low quadrupole***

$v_k = \delta\Psi_k a(t)\dot{\phi}/H$ **obeys a wave eqn. in conformal time** η

during slow roll

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2}\right] v_k = 0 \; ; \nu = \frac{3}{2} + 3\epsilon_v - \eta_v$$

Solutions with Bunch-Davies b.c.

$$g_\nu(k;\eta) = \frac{1}{2}\, i^{\nu+\frac{1}{2}} \sqrt{-\pi\eta}\, H_\nu^{(1)}(-k\eta) \; ; \; g_\nu(k;\eta) \overset{\eta\to-\infty}{=} \frac{1}{\sqrt{2\,k}}\, e^{-ik\eta}$$

General solution: $S(k;\eta) = A(k)\, g_\nu(k;\eta) + B(k)\, [g_\nu(k;\eta)]^*$

With normalization condition $|A(k)|^2 - |B(k)|^2 = 1$

Quantization:

$$v_k(\eta) = a_k S(k;\eta) + a_k^\dagger S^*(k;\eta)$$

B(k)=0 → B.D. vacuum

Power spectrum

$$\mathcal{P}(k) = \langle 0 || \delta\Psi_k|^2 | 0 \rangle = \mathcal{P}_{BD}(k)\, [1 + D(k)]$$

$$\mathcal{P}_{BD}(k) = \frac{H^2}{\epsilon_v M_P^2} \left(\frac{k}{k_0} \right)^{n_s - 1}$$

$D(k)$: **Transfer function for boundary conditions**

$$n_s = 1 - 6\epsilon_v + 2\eta_v$$

Conditions on D(k):

- **Negligible backreaction on Einstein's eqns.**
- **Finite $T_{\mu\nu}$**
- **Renormalization of $T_{\mu\nu}$ with counterterms indep. of b.c**

→ **D(k) falls faster than $1/k^4$**

Change in b.c. → **change in C_l**

$$\frac{\Delta C_l}{C_l} = \frac{\int_0^\infty D(\kappa\, x)\ f_l(x)\, dx}{\int_0^\infty f_l(x)\, dx}$$

$$\kappa = \frac{a_0 H_0}{3.3}$$

$$f_l(x) = x^{n_s - 2}[j_l(x)]^2$$

Rapid fall-off of D(k) → ***ONLY LOW MULTIPOLES ARE AFFECTED***

Q: What is the origin of D(k)?? (what determines the b.c.??)

A: a fast-roll stage PRIOR to slow roll

Allowing for ***RAPID*** variation of the condensate ϕ

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2} - \mathcal{V}(\eta)\right] v_k = 0$$

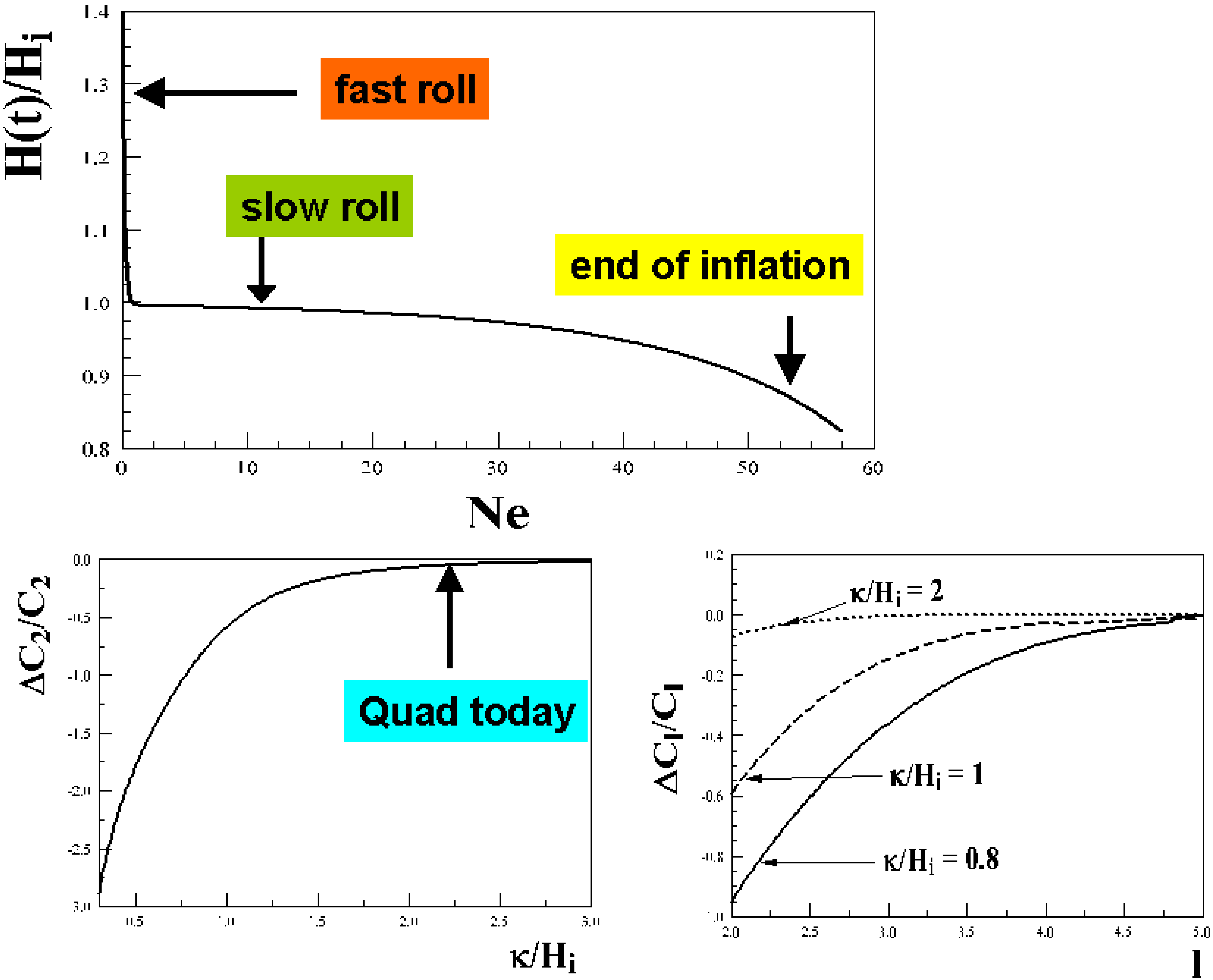
↓

Depends on high(er) derivatives of ϕ : ***neglible in slow roll***, ***large for fast roll***

WHEN? large INITIAL $\dot{\phi}$ but large FRICTION term → short fast roll stage

→ $\mathcal{V}(\eta) =$ ***LOCALIZED POTENTIAL***

$D(k) \propto T(k)$ = Transmission coeff. of SCATTERING PBM!!


fast roll
slow roll
end of inflation
H(t)/H_i
Ne
Quad today
ΔC_2/C_2
κ/H_i
κ/H_i = 2
κ/H_i = 1
κ/H_i = 0.8
ΔC_l/C_l
l

Executive summary:

- Initial conditions: set by a *fast roll* stage prior to slow roll
- Fast roll ~ *GENERIC* initial condition with kin. ~pot. energy
- Fast roll: localized *attractive* potential in mode equations
- D(k) = transmission coeff. = transfer function
- Suppression of low multipoles

Analysis: fast roll ~ 2-3 e-folds, IF k_Q crosses horizon ~ 2-3 e-folds after beginning of slow roll ~ 15-20% suppression!

Quad. suppression mechanism *WITHIN* eff. field theory

New vs. chaotic inflation and reconstruction program: confronting *WMAP 3*

Implement eff. field theory + slow roll as 1/Ne expansion to systematically explore a large ``family'' of inflaton potentials.

WMAP 3 + LSS:

$$n_s = 0.958 \pm 0.016 \qquad \text{(assuming } r = 0 \text{ with no running)}$$

$$r < 0.28 \ (95\% \, CL) \quad \text{no running}$$

$$r < 0.67 \ (95\% \, CL) \quad \text{with running}$$

<u>Slow Roll expansion</u>: a hierarchy of dimensionless parameters:

$$\epsilon_v = \frac{M_P^2}{2} \left[\frac{V^{'}(\phi)}{V(\phi)} \right]^2 \quad , \quad \eta_v = M_P^2 \, \frac{V^{''}(\phi)}{V(\phi)} \cdots$$

as a 1/Ne expansion:

$$N\left[\phi(t)\right] = -\frac{1}{M_P^2} \int_{\phi(t)}^{\phi_{end}} V(\phi) \, \frac{d\phi}{dV} \, d\phi$$

$$\phi = \sqrt{N_e} M_P \, \chi \longleftarrow$$ **Rescale field**

$$V(\phi) = N_e \, M^4 \, w(\chi)$$

$w(\chi) \longrightarrow \sim \mathcal{O}(1)$

$M^4 \longrightarrow$ **energy scale of inflation**

$$1 = -\int_{\chi_c}^{\chi_{end}} \frac{w(\chi)}{w'(\chi)}\, d\chi$$

value of χ at the END of inflation

value of χ Ne e-folds before END of inflation

Independent of Ne: an implicit relation between χ and couplings

$$\epsilon_v = \frac{1}{2\,N_e}\left[\frac{w'(\chi_c)}{w(\chi_c)}\right]^2 \quad , \quad \eta_v = \frac{1}{N_e}\,\frac{w''(\chi_c)}{w(\chi_c)}$$

Explicit dependence on Ne: SIMPLE RESCALING

$$n_s - 1 = -6\epsilon_v + 2\eta_v \sim 1/N_e$$

$$r = 16\epsilon_v \sim 1/N_e$$

$$\frac{dn_s}{d\ln k} = -\frac{2}{N_e^2}\left\{\frac{w'(\chi_c)w'''(\chi_c)}{w^2(\chi_c)} + 3\left[\frac{w'(\chi_c)}{w(\chi_c)}\right]^4 - 4\,\frac{[w'(\chi_c)]^2\, w''(\chi_c)}{w^3(\chi_c)}\right\}$$

Simple scaling with 1/N_e, choose N = 50 as representative

Family of potentials

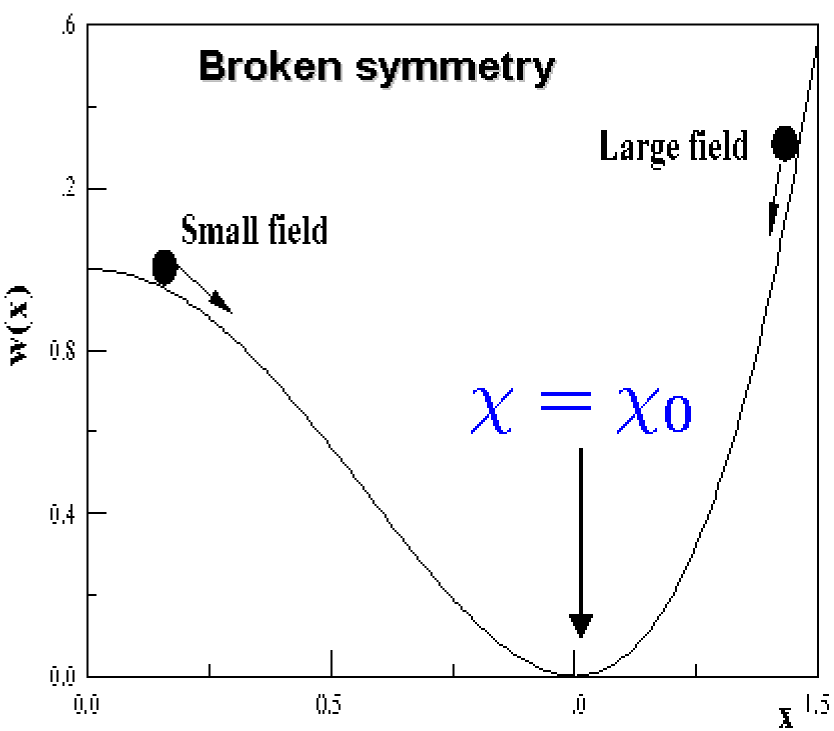
$$V(\phi) = V_0 - \frac{1}{2}\, m^2\, \phi^2 + \frac{\lambda}{2\, n}\, \phi^{2\, n} \quad \text{, broken symmetry}$$

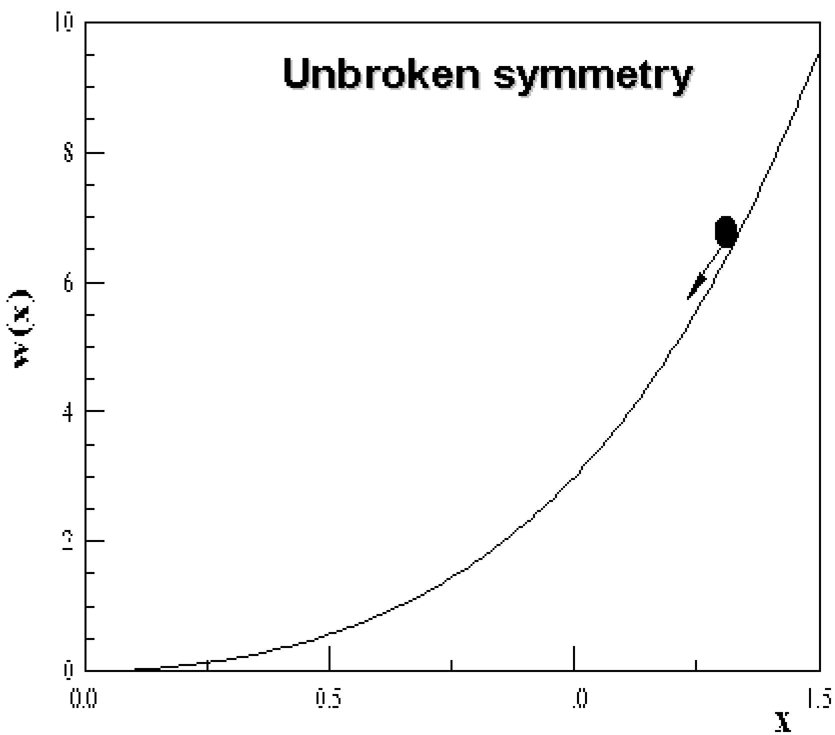
$$V(\phi) = \frac{1}{2}\, m^2\, \phi^2 + \frac{\lambda}{2\, n}\, \phi^{2\, n} \quad \text{, unbroken symmetry}$$

Rescale fields and couplings:

$$\lambda = \frac{m^2\, g}{M_{Pl}^{2n-2}\; N_e^{n-1}} \; ; \; g = \frac{1}{\chi_0^{2n-2}} \; ; \; x = \frac{\chi}{\chi_0}$$

$$w(\chi) = \frac{\chi_0^2}{2n}\left[n\,(1-x^2) + x^{2n} - 1\right] \qquad w(\chi) = \frac{\chi_0^2}{2n}\left[n\,x^2 + x^{2n}\right]$$





Broken Symmetry

$$\frac{2\,n}{\chi_0^2} = \int_X^1 \frac{dx}{x}\,\frac{n\,(1-x^2)+x^{2\,n}-1}{1-x^{2\,n-2}}$$

Unbroken symmetry

$$\frac{2\,n}{\chi_0^2} = \int_0^X \frac{n+x^{2\,n-2}}{1+x^{2\,n-2}}\;x\;dx \qquad X = \frac{\chi_c}{\chi_0}$$

Conditions for number of e-folds

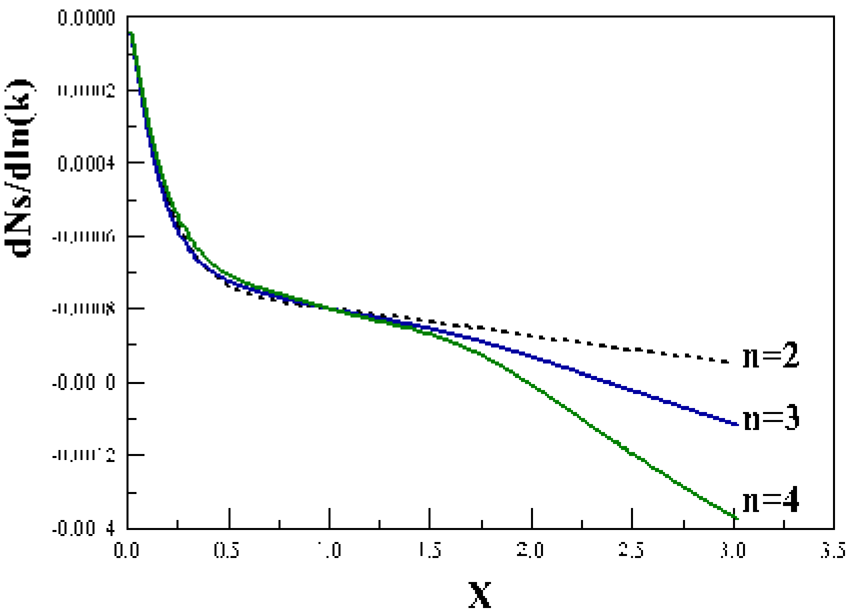
- X determines the value of the field Ne e-folds before the end of inflation
- χ_0 determines the value of the coupling for which the value of X is Ne before the end of inflation

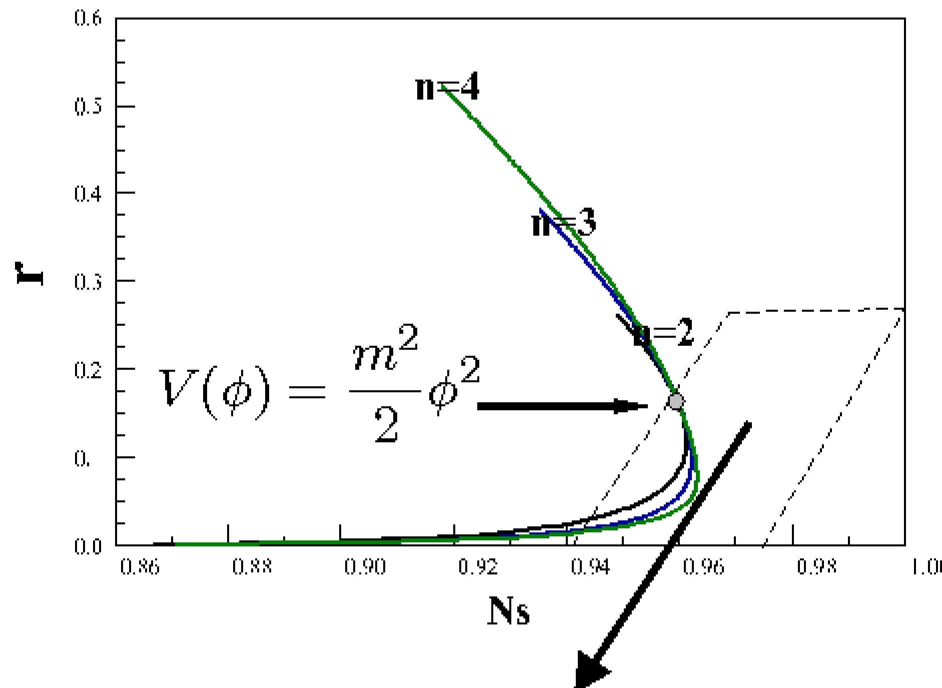
STRATEGY

1) Vary X, find χ_0 construct w[x] and derivatives
2) Find ϵ_v , η_v, n_s, r, $dn_s/d\ln k$ as a function of X
3) Plot parametrically

RESULTS

1) New inflation (B.S.) Ne= 50 (change accordingly)



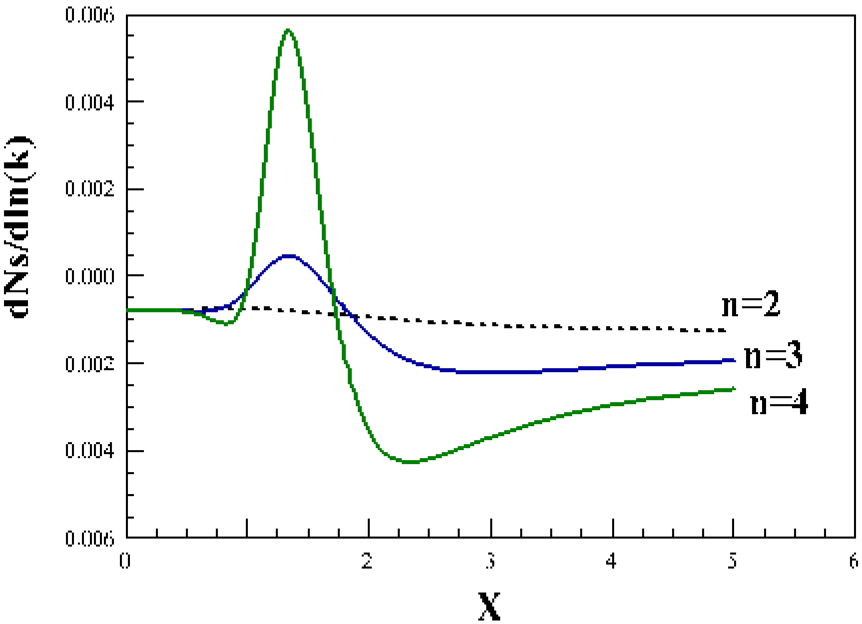


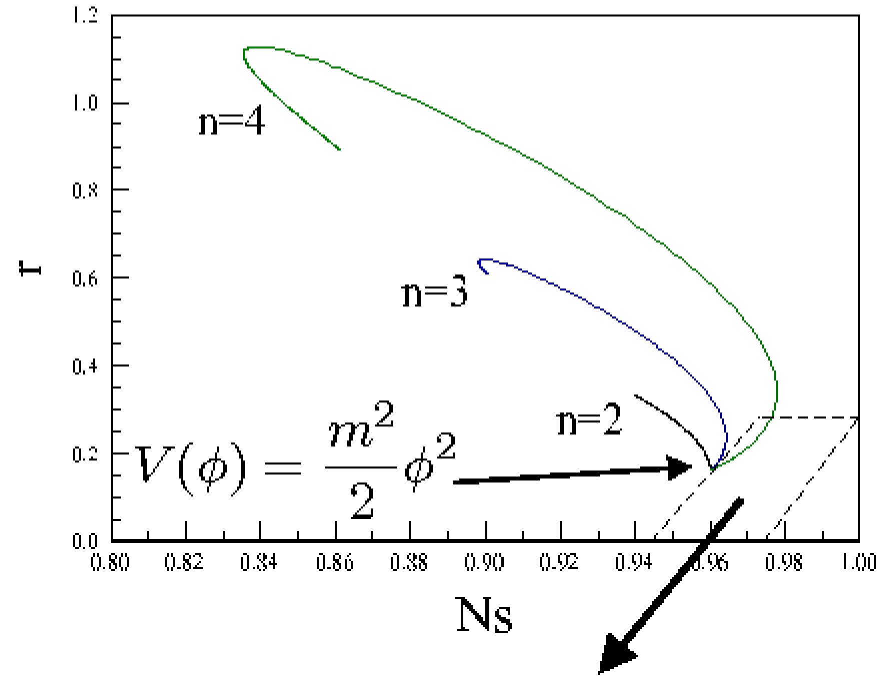
WMAP 3 marginalized region of r- n_s (95% CL)

*

Large region of consistency for small field New Inflation

2) Chaotic inflation



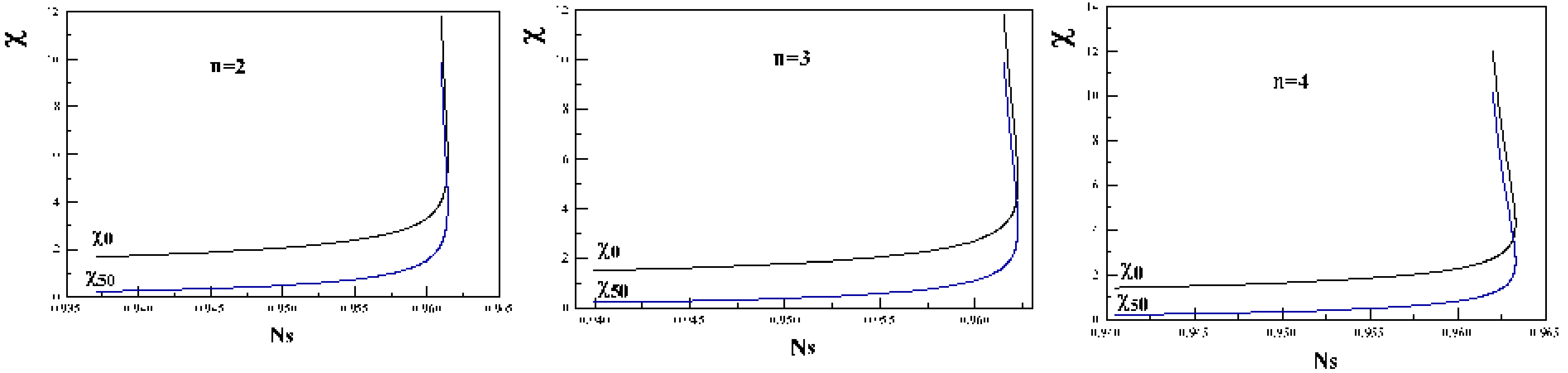


WMAP 3 marginalized region of r- ns (95% CL)

* Small region of consistency with WMAP 3

FIELD RECONSTRUCTION: Ne =50

New inflation small field: region of consistency with WMAP 3



Symmetry breaking scale : $\phi_0 \sim 10 M_P$

Crossing scale: $\phi_{50} \sim M_P$

FINAL—FINAL SUMMARY AND CONCLUSIONS

- ✓ Effective field theory H/Mp <<1, 1/Ne-slow roll expansion robust, systematic, predictive
- ✓ Quantum corrections suppressed by (H/Mp)^2
- ✓ Fast roll stage prior to slow roll ⟶ modifies b.c. for scalar perturbations ⟶ quadrupole suppression ~ 15-20% for total Ne ~ 55.
- ✓ 1/Ne expansion ⟶ systematic exploration of family of inflaton potentials+ field reconstruction .
- ✓ Small field New Inflation larger region of consistency with WMAP3+LSS data.
- ✓ Potentials with larger overlap with marginalized WMAP 3 data symmetry breaking scale ~ 10 Mp, crossing scale ~ Mp.