M. Giovannini, PRD 73, 101302 (2006); PRD 123507 (2004); PRD 56, 3198 (1997);
M. Giovannini + N. Q. Lan, arXiv: 0905.3515 (PRD in press);
M. Giovannini + K. Kunze, PRD 063007 (2009);
M. Giovannini, PRD 79 121302 (2009);
PRD 103007 (2009);
arXiv: 0907.3235.

Large-scale magnetic fields in the standard model

Massimo Giovannini (CERN & INFN)

Paris, July 2009

Standard Model

a) For L > 300 million light years : Universe homogeneous and isotropic;

b) Sources driving the evolution of the geometry: perfect (barotropic) fluids;

c) Dynamical law connecting the sources to the geometry: GR;

a) FLRW line element:

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right]$$

b) Adiabatic evolution:

$$T^{\nu}_{\mu} = (p+\rho)u_{\mu}u^{\nu} - p\delta^{\nu}_{\mu} \qquad p = w\rho$$

$$\nabla_{\mu}T^{\mu}_{\nu} = 0 \Rightarrow$$ Entropy conservation

c) Friedmann-Lemaitre equations :

$$R^{\nu}_{\mu} - \frac{1}{2}\delta^{\nu}_{\mu}R = 8\pi G T^{\nu}_{\mu}$$

$$H^2 = \frac{8\pi G}{3}\rho_t - \frac{k}{a^2}$$

$$\dot{H} = -4\pi G(\rho_t + p_t) + \frac{k}{a^2}$$

$$\dot{\rho}_t + 3H(\rho_t + p_t) = 0$$

$$dt = a(\tau)d\tau$$

$$ds^2 = a^2(\tau)[d\tau^2 - \gamma_{ij}dx^i dx^j]$$

$$\mathcal{H}^2 = \frac{8\pi G}{3}\rho_t a^2 - k$$

$$\mathcal{H}^2 - \mathcal{H}' = 4\pi G(\rho_t + p_t)a^2 + k$$

$$\rho_t' + 3\mathcal{H}(\rho_t + p_t) = 0$$

$$H = \frac{\dot{a}}{a} = \frac{d\ln a(t)}{dt}$$

Cosmic time parametrization

$$\mathcal{H} = \frac{a'}{a} = \frac{d\ln a(\tau)}{d\tau}$$

Conformal time parametrization

LambdaCDM paradigm

LambdaCDM

$$(\Omega_{b0},\ \Omega_{c0},\ \Omega_\Lambda,\ h_0,\ n_s,\ \tau) = (0.0441,\ 0.214,\ 0.742,\ 0.719,\ 0.963,\ 0.087)$$

WAP 5-year alone

$$(\Omega_{b0},\ \Omega_{c0},\ \Omega_\Lambda,\ h_0,\ n_s,\ \tau) = (0.042,\ 0.198,\ 0.76,\ 0.732,\ 0.958,\ 0.089).$$

WAP 3-year alone

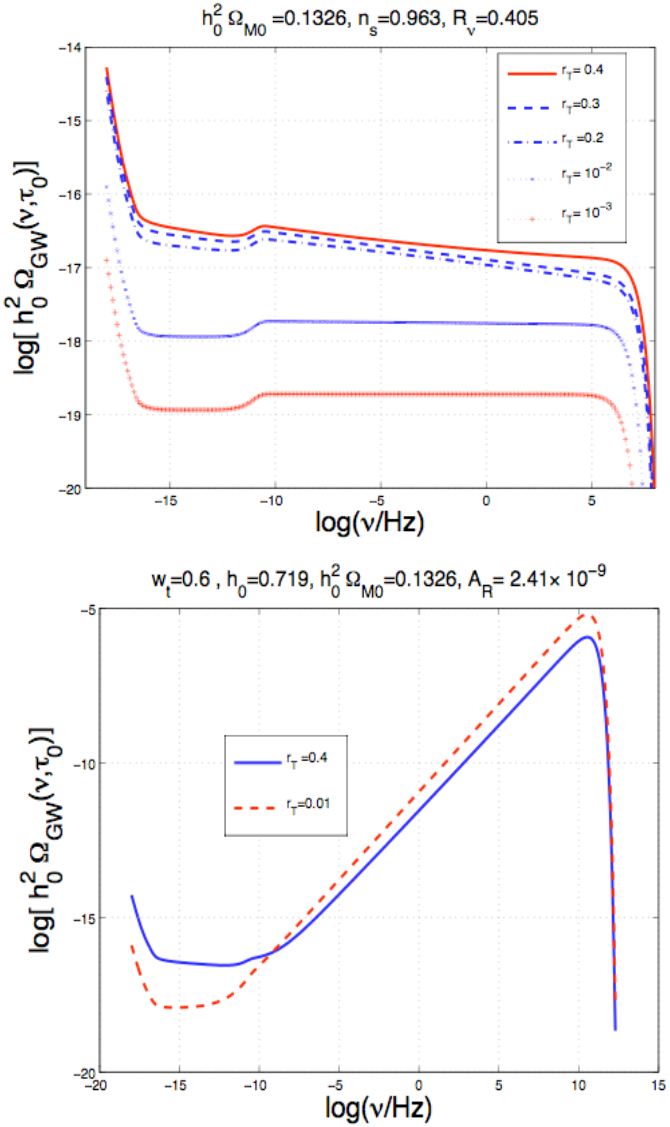
Massive neutrinos?

wCDM ?
(dark energy fluctuates)

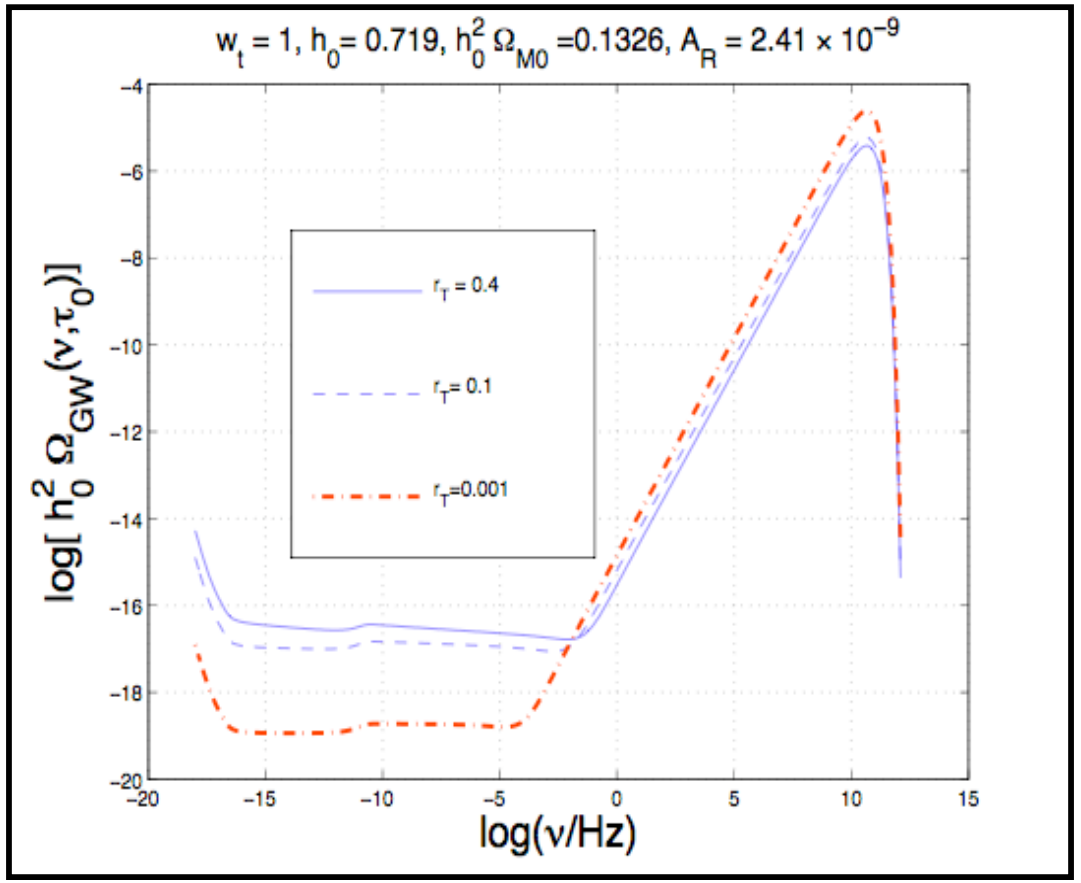
Tensors?

Data	r_T	n_s	α_S	Ω_Λ	Ω_{M0}	k_{eq}Mpc
WMAP5 alone	< 0.43	0.986 ± 0.22	0	$0.770^{+0.033}_{-0.032}$	$0.230^{0.032}_{-0.033}$	0.00936
WMAP5 + Acbar	< 0.40	$0.985^{0.019}_{-0.020}$	0	0.767 ± 0.032	0.233 ± 0.032	0.00944
WMAP5+ LSS + SN	< 0.20	0.968 ± 0.015	0	0.725 ± 0.015	0.275 ± 0.015	0.00999
WMAP5+ CMB data	< 0.36	0.979 ± 0.020	0	0.775 ± 0.032	0.225 ± 0.032	0.00922

Tensors + modified thermal history



M. Giovannini, Phys. Lett. B 668, 44 (2008)
Class Quantum Grav 045004 (2009)



A Magnetized Universe

- Large-scale magnetic fields (typical length-scales > 1 A.U.) $1 A.U. = 1.49\ 10^{13}\ cm$

- First speculations: early forties (Alfven) late forties (Fermi, Fermi & Chandrasekar) on cosmic ray physics

$1\ \mu G = 0.1\ nT = 10^{-26}\ GeV^2$

-Today: magnetic fields measured with various techniques

Zeeman splitting of radio transitions

$$\Delta\nu_Z = \frac{e\bar{B}_\parallel}{2\pi m_e}$$

$$\Delta\nu_{Doppler} \simeq \left(\frac{v_{th}}{c}\right)\nu \gg \Delta\nu_{Zeeman} \simeq \frac{e\bar{B}_\parallel}{2\pi m_e}$$

Synchrotron emission

$$\varepsilon(\nu) = 10^{-23}\, n_{er0}\, L\, \xi(\gamma)\, (6.3\times 10^{18})^{(\gamma-1)/2} (B_\perp)^{(\gamma+1)/2}\, \nu^{(1-\gamma)/2}\ \ erg\ sec^{-1}\ cm^{-2}\ Hz^{-1}$$

Faraday rotation

$$\Delta\,\phi = \frac{f_e}{2}\left(\frac{\omega_p}{\omega}\right)^2 \omega_B\, \Delta z$$

$$\omega_p = \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2} \qquad \omega_B = \frac{eB}{mc}$$

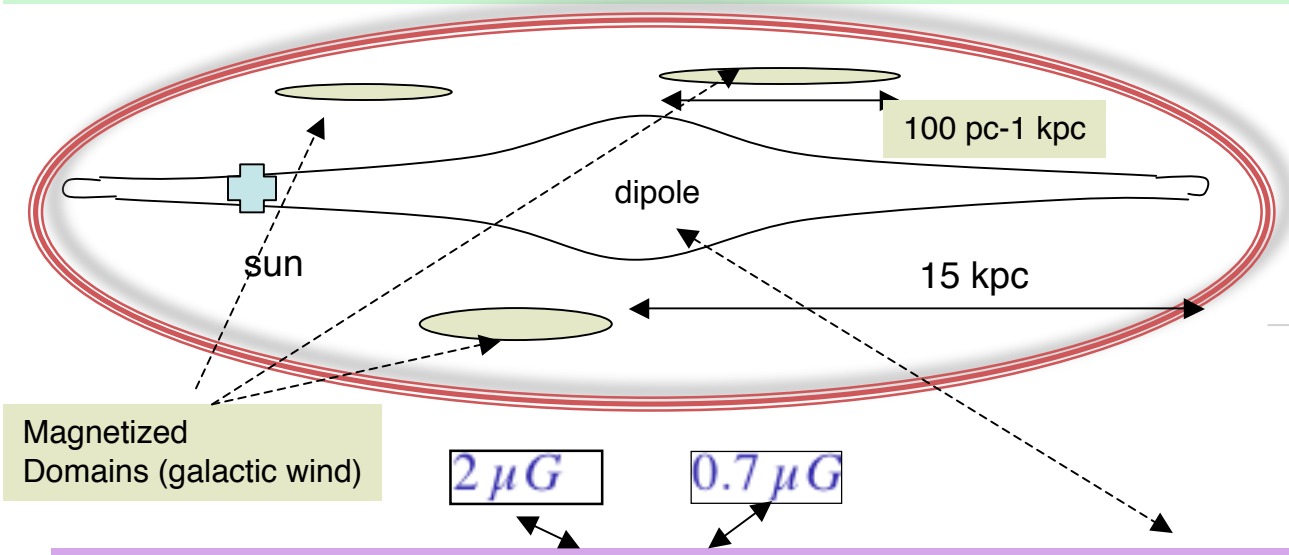
$$\phi = RM\,\lambda^2 + \phi_0$$

$$RM = \frac{\Delta\phi}{\Delta\lambda^2} = 811.9 \int \left(\frac{n_e}{cm^{-3}}\right)\left(\frac{B_\parallel}{\mu G}\right) d\left(\frac{\ell}{kpc}\right)\ \frac{rad}{m^2}$$

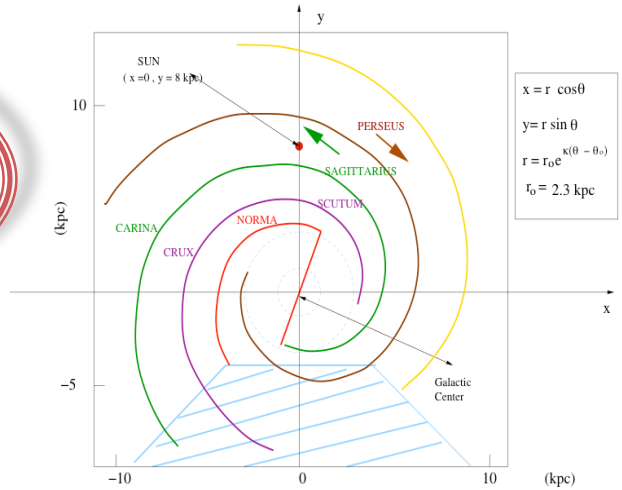
$$DM \propto \int n_e d\ell$$

$$\langle B_\parallel \rangle = \frac{RM}{DM}$$

Galaxies, clusters



SGP survey
Parkes



Milky way: toroidal field south/north hemisphere; dipoles (center) $10\ rad/m^2$

Local Group: Andromeda, Magellanic Clouds,… $2 - 7\ \mu G$ (elliptical galaxies: shorter scale)

Abell Clusters (like COMA): magnetic fields inside cluster (VLA+ROSAT) [Faraday RM]
Typical RM: $100\ rad/m^2$ $B \sim 0.5\ \mu G = 500\ nG$ $L \sim 50 - 100\ kpc$

Hercules / Perseus-Pisces $B_L \simeq 0.5 \mu G$

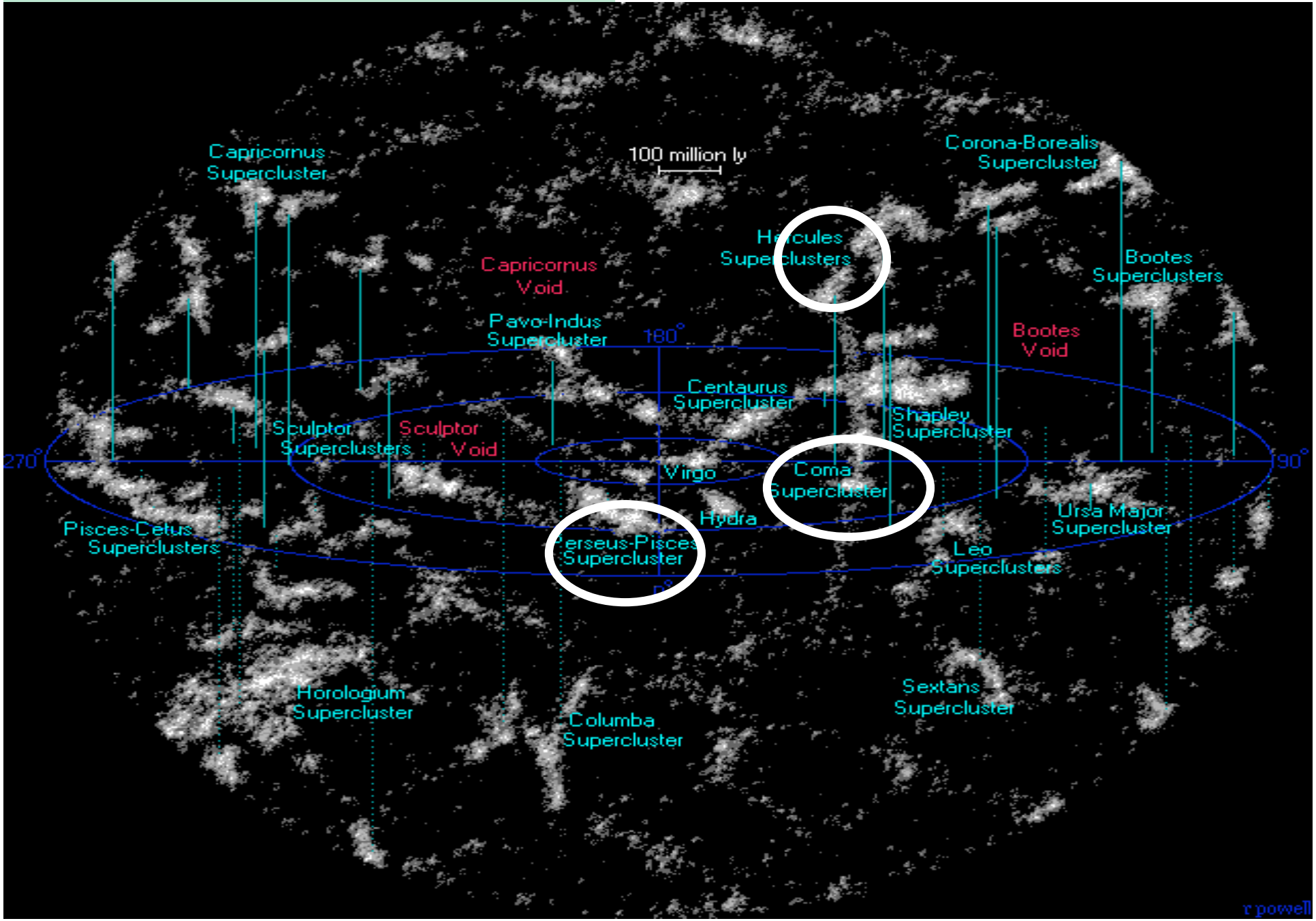
$n_e \simeq 10^{-6}\ cm^{-3}$ GRG $L \simeq 500\ kpc$

Faraday rotation should be reduced as $(z+1)^{-2}$

Kronberg, Bernet, Minati,Lilly
Short, Higdom arXiv. 07120435

High redshift quasars (up to z ~ 3.7)

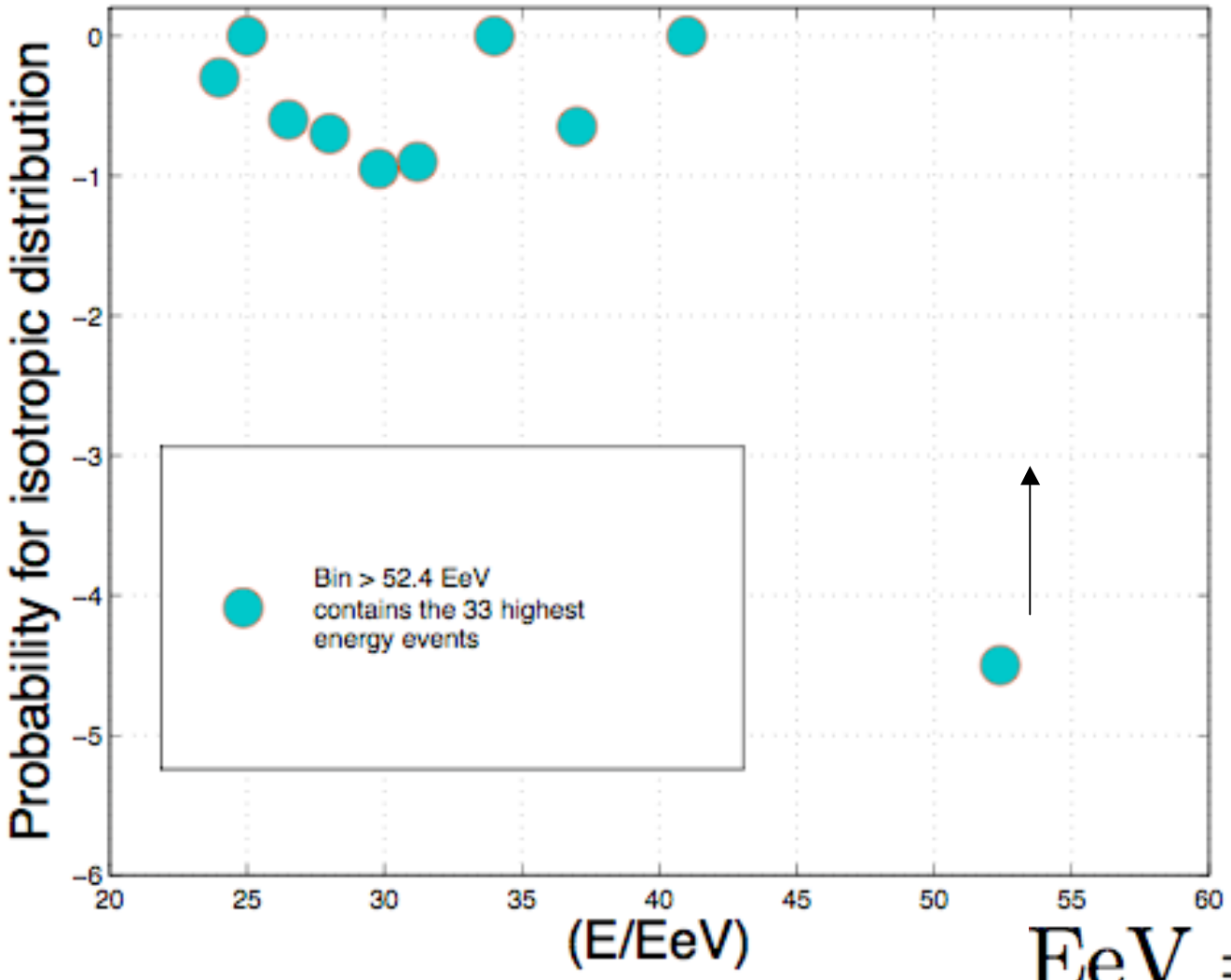
Superclusters



Auger/1

$$\frac{\delta}{\deg} \simeq 3\left(\frac{B}{3\mu\mathrm{G}}\right)\left(\frac{L}{\mathrm{kpc}}\right)\left(\frac{6\times10^{19}\,\mathrm{eV}}{E}\right)z$$

$$\frac{E}{Z} > 10^{19}\mathrm{eV}$$



Cocoon~ 75 Mpc

$$\mathrm{EeV} = 10^{18}\,\mathrm{eV}$$

Auger/2

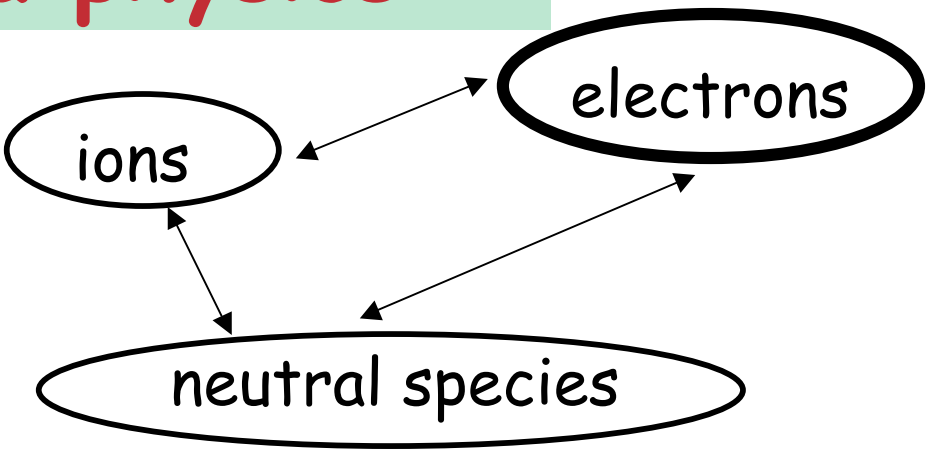
$$\psi_{\max} = 3.1\,\mathrm{deg} \qquad z_{\max} = 0.018$$

$$E_{\max} = 55\ \mathrm{EeV}$$

Period	Exposure	GP	N	k	k_{iso}	P
I (1 January 2004- 26 may 2006) exploratory	4390	unmasked	14	9	2.9	
II (27 may 2006- 31 August 2007)	4500	unmasked	13	9	2.5	2×10^{-4}
III (1 september 2007- 31 March 2009)	8150	unmasked	31	8	6.5	0.33
II+ III	12650	unmasked	44	17	9.2	6×10^{-3}

17 out of 44 events that correlate in the independent data (roughly 2.5 sigma from the value expected from purely isotropic distribution)

Plasma physics



Tenets of plasma physics:
1) Kinetic descriptions : phase space distribution (PSD)
2) Multi-fluid description : moments of PSD
3) Effective variables (e.g. total current & baryon velocity)

(Topological) conservation laws

$$\frac{d}{dt}\mathcal{H}_M = -\frac{1}{4\pi\sigma}\int_V d^3x\ \vec{B}\cdot\vec{\nabla}\times\vec{B},$$

Magnetic helicity

$$\mathcal{H}_M = \int_V d^3x \vec{A}\ \cdot\vec{B},$$

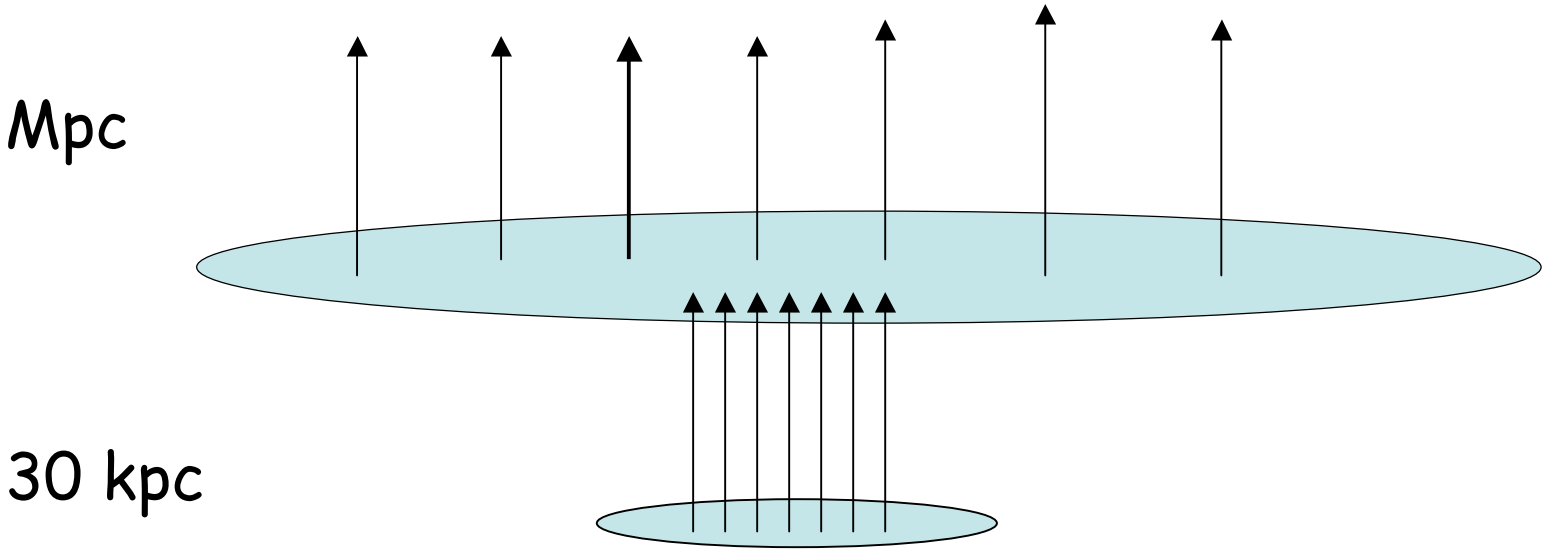
$$\frac{d}{dt}\int_\Sigma \vec{B}\cdot d\vec{\Sigma} = -\frac{1}{4\pi\sigma}\int_\Sigma \vec{\nabla}\times(\vec{\nabla}\times\vec{B})\cdot d\vec{\Sigma},$$

Magnetic flux

Dynamos & compressional amplification(s)

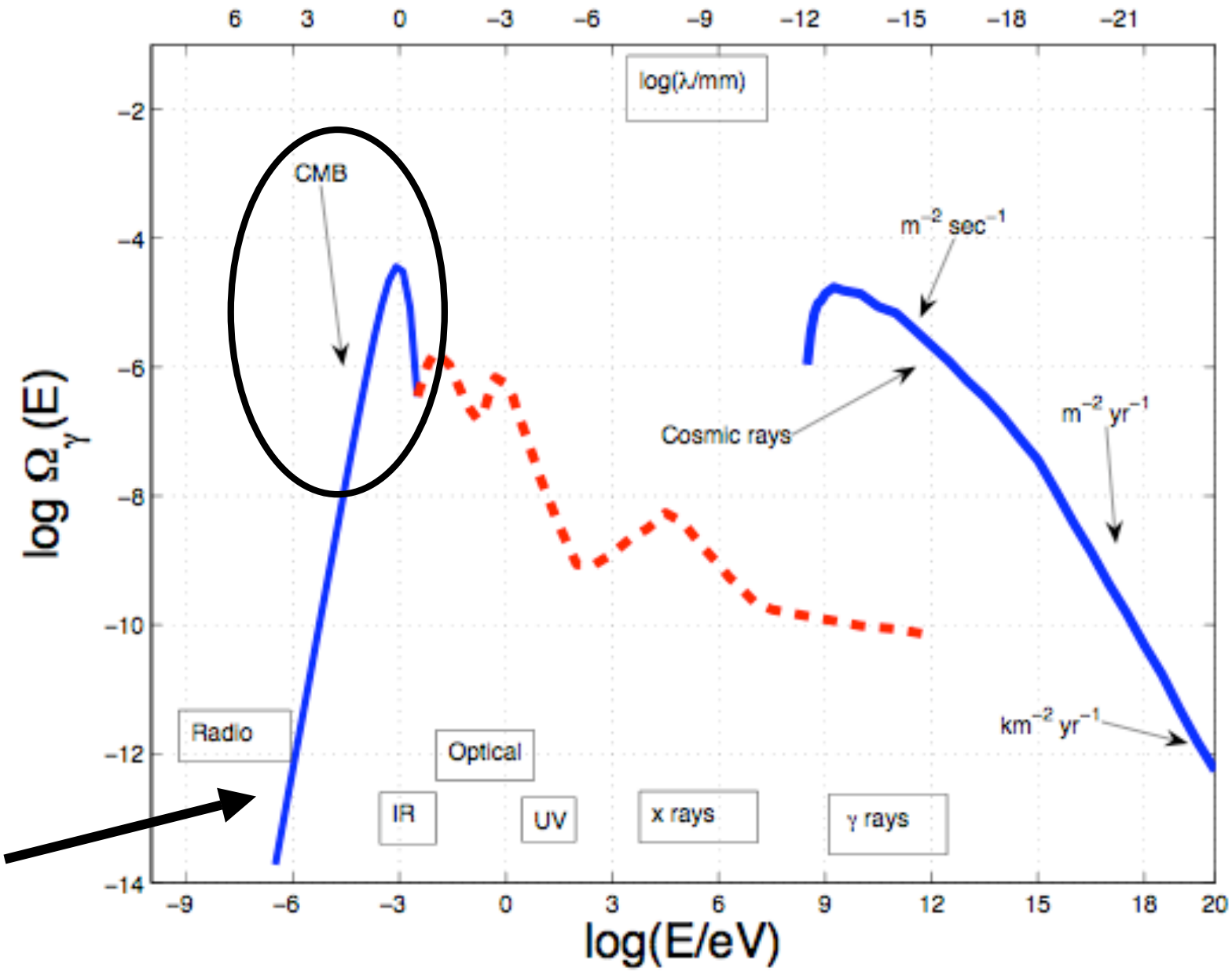
$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \frac{1}{4\pi\sigma} \nabla^2 \vec{B}$$

Kinetic energy is transferred to magnetic energy



Cascades ? Direct? Inverse?

Magnetic fields and CMB spectrum



Magnetized CMB anisotropies

1) large-scale magnetic fields DO NOT break spatial isotropy;
2) curved-space effects described within GR;
3) plasma dynamics : exactly like in terrestrial laboratories;

From (1)

$$\langle B_i(\vec{k},\tau)B_j(\vec{p},\tau)\rangle = \frac{2\pi^2}{k^3}P_{ij}(k)\mathcal{P}_B(k)\delta^{(3)}(\vec{k}+\vec{p})$$

From (2)

All the evolution equations+ initial conditions (very important!)

From (3)

Plasma hierarchies

a) Debye scale << Coulomb mean free path;
b) Coulomb rate << plasma frequency;
c) Larmor radius << inhomogeneity scale of the MF

Plasma hierarchies

Plasma globally neutral ->

$$n_e = n_i = n_0 = \eta_{b0}\, n_{\gamma 0}$$

Plasma hierarchies controlled by PLASMA PARAMETER

$$g_{plasma} = \frac{1}{V_D n_0 x_e} = 2.308 \times 10^{-7} \sqrt{x_e} \left(\frac{h_0^2 \Omega_{b0}}{0.02273} \right)^{1/2}$$

$$\eta_{b0} = 6.219 \times 10^{-10} \left(\frac{h_0^2 \Omega_{b0}}{0.02773} \right) \left(\frac{T_{\gamma 0}}{2.725} \right)^{-3}$$

$$V_D = \tfrac{4}{3}\pi \lambda_D^3 \longleftrightarrow \lambda_D = \sqrt{\frac{T}{8\pi e^2 n_0 x_e}}$$

1) $\frac{\lambda_D}{\lambda_{Coul}} = \frac{g_{plasma}}{48\pi} \ln \Lambda_C, \qquad \Lambda_C = \frac{18\sqrt{2}}{g_{plasma}}$ Debye scale << Coulomb mean free path

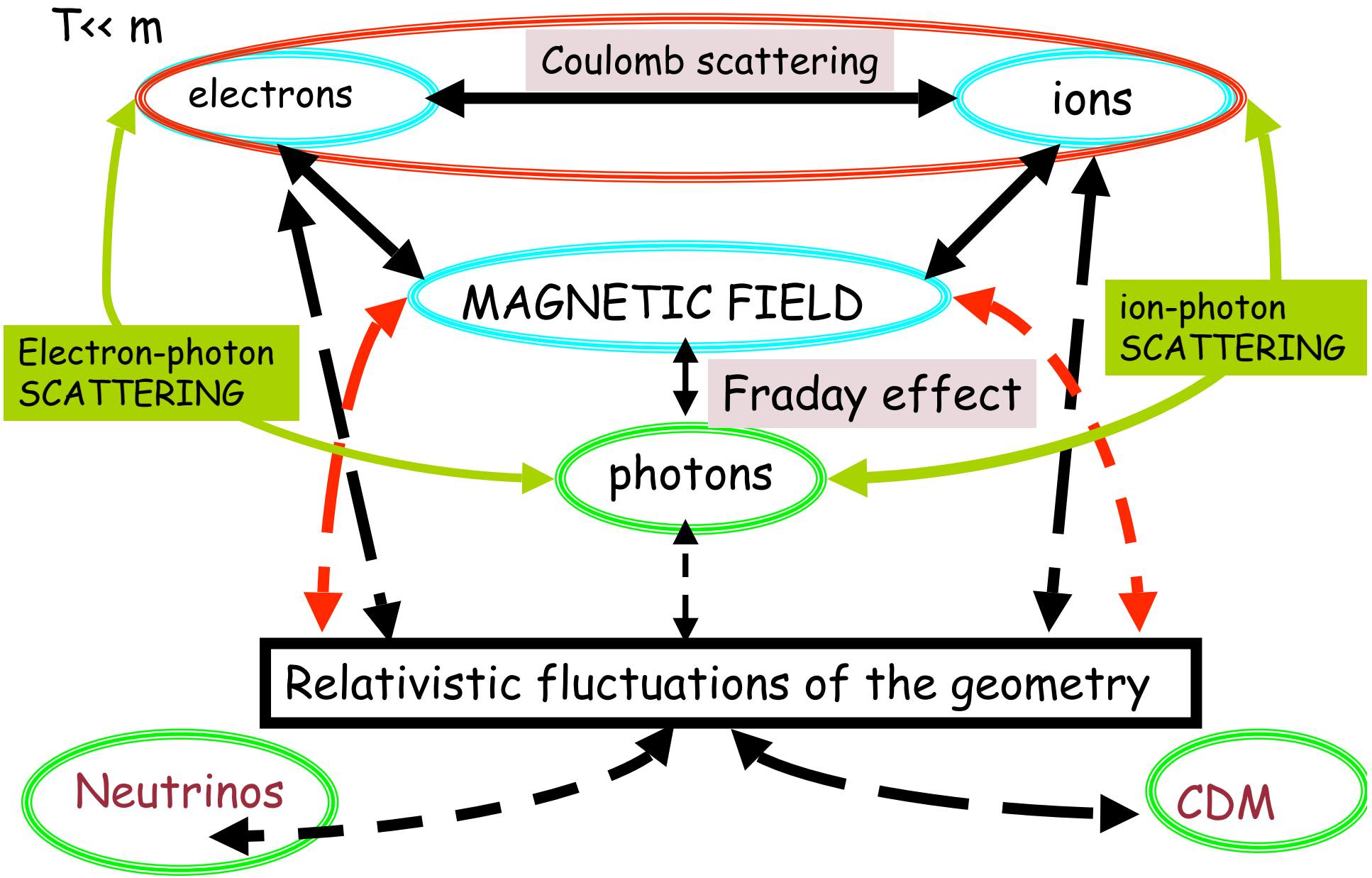
2) $\frac{\Gamma_{Coul}}{\omega_{pe}} = \frac{\ln \Lambda_C}{24\sqrt{2}\pi} g_{plasma}, \qquad \omega_{pe} = \sqrt{\frac{4\pi n_0 x_e}{m_e}}.$ Coulomb rate<< plasma freq.

3) $\sigma = \frac{\omega_{pe}^2}{4\pi \Gamma_{Coul}} = \frac{6\sqrt{2}}{\ln \Lambda_C} \frac{\omega_{pe}}{g_{plasma}}.$ Conductivity very large in units of pl. freq

4) $L_\sigma \simeq (4\pi\sigma H_{eq})^{-1}, \qquad \sigma = \frac{T}{e^2 \ln \Lambda_C} \left(\frac{T}{m_e} \right)^{1/2},$ Larmor radius much smaller Than inhomogeneity scale of the field

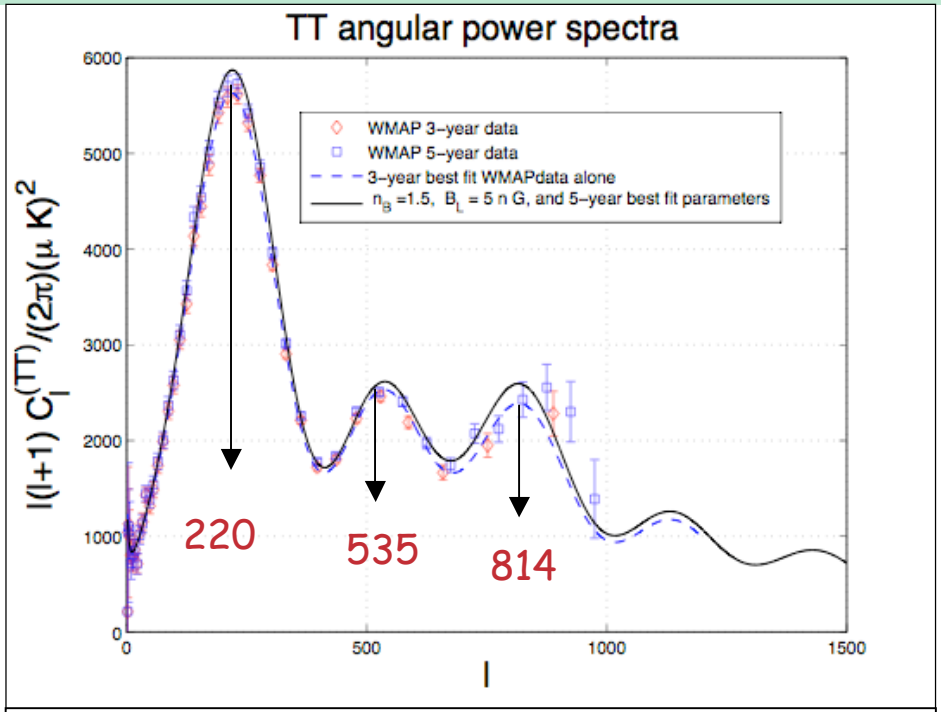
5) $r_{Be} \ll L \simeq r_H, \qquad r_{Be} = \frac{v_\perp}{\omega_{Be}}, \qquad v_\perp \simeq v_{th}$

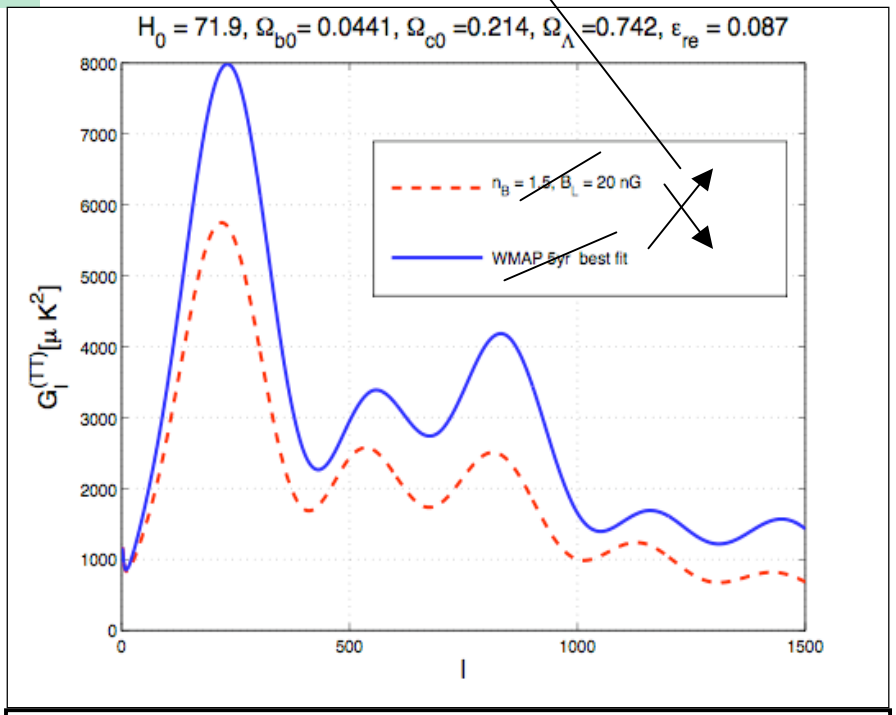
Magnetized (cold) plasma

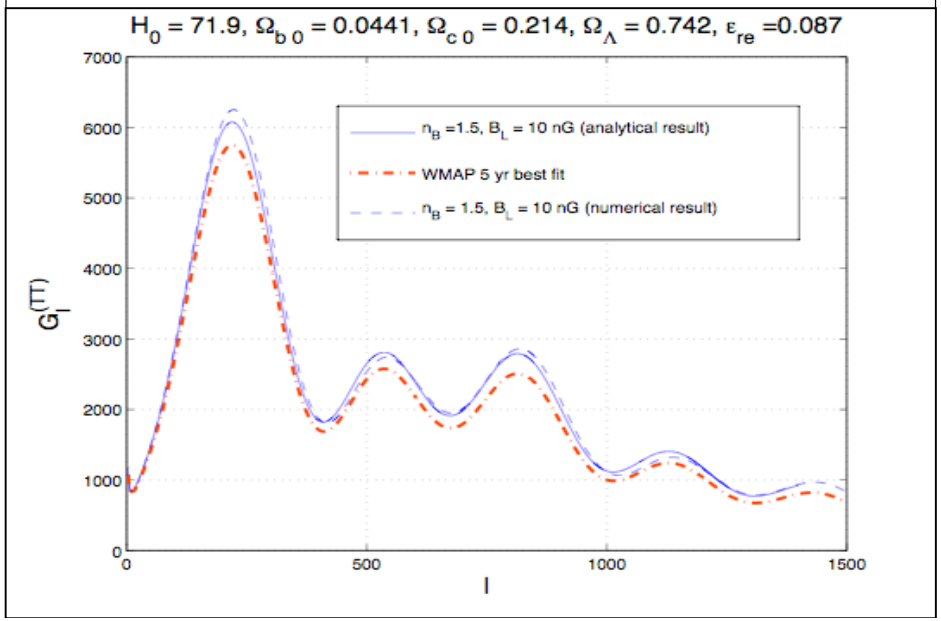


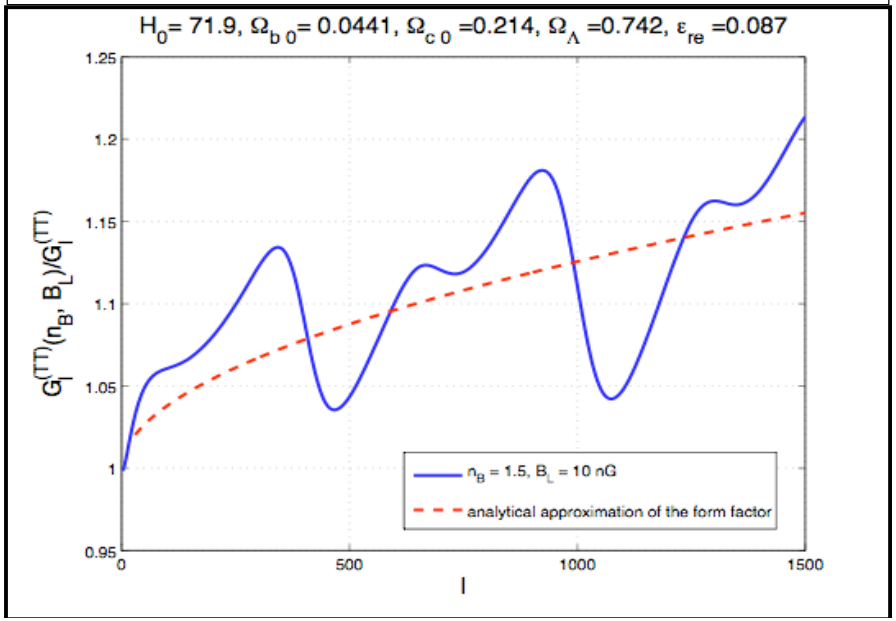
Magnetized TT correlations

M. G. PRD 103007 (2009).



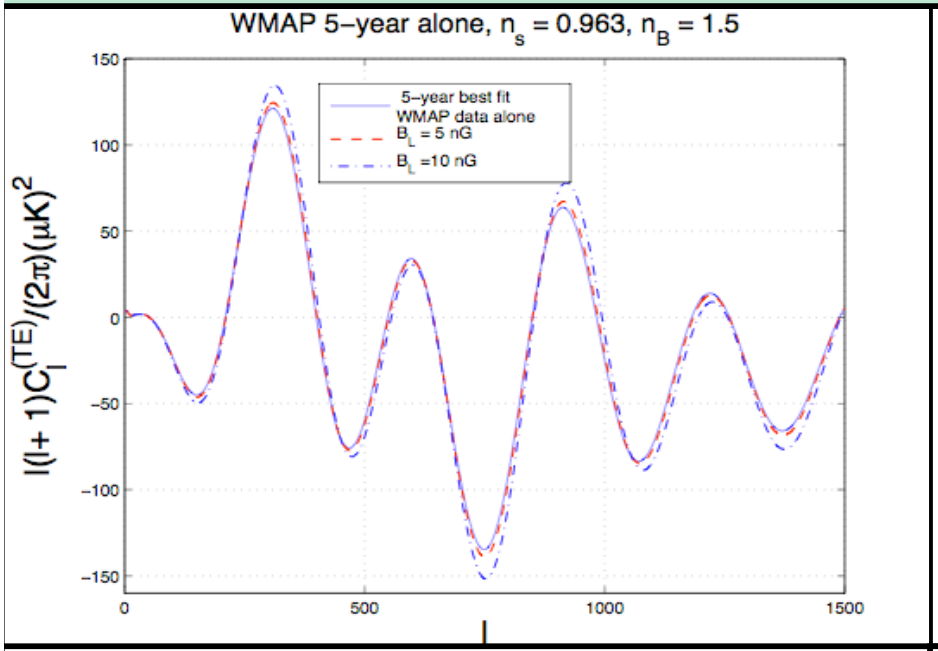


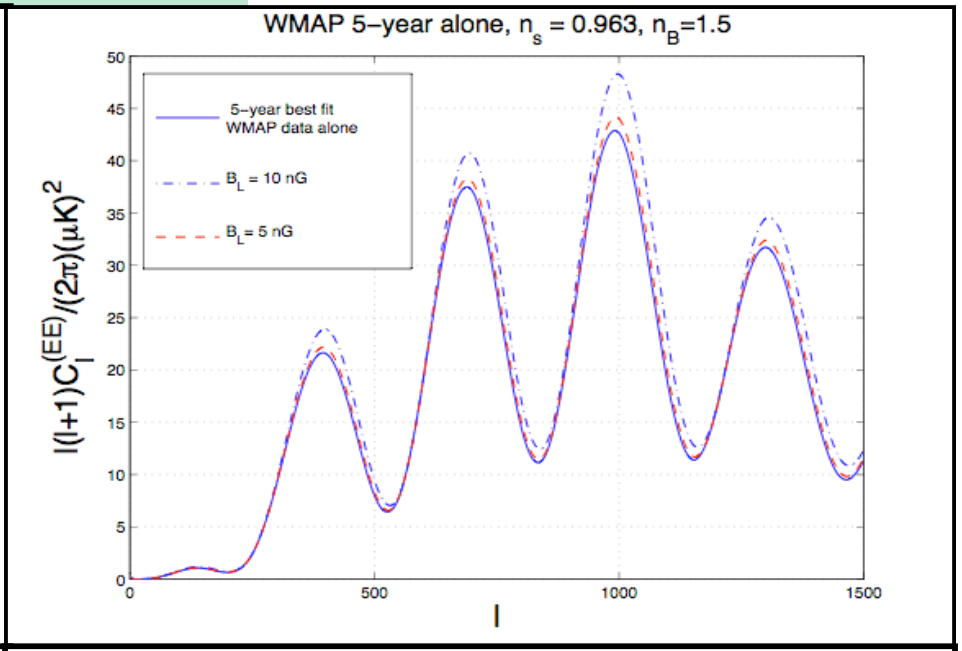


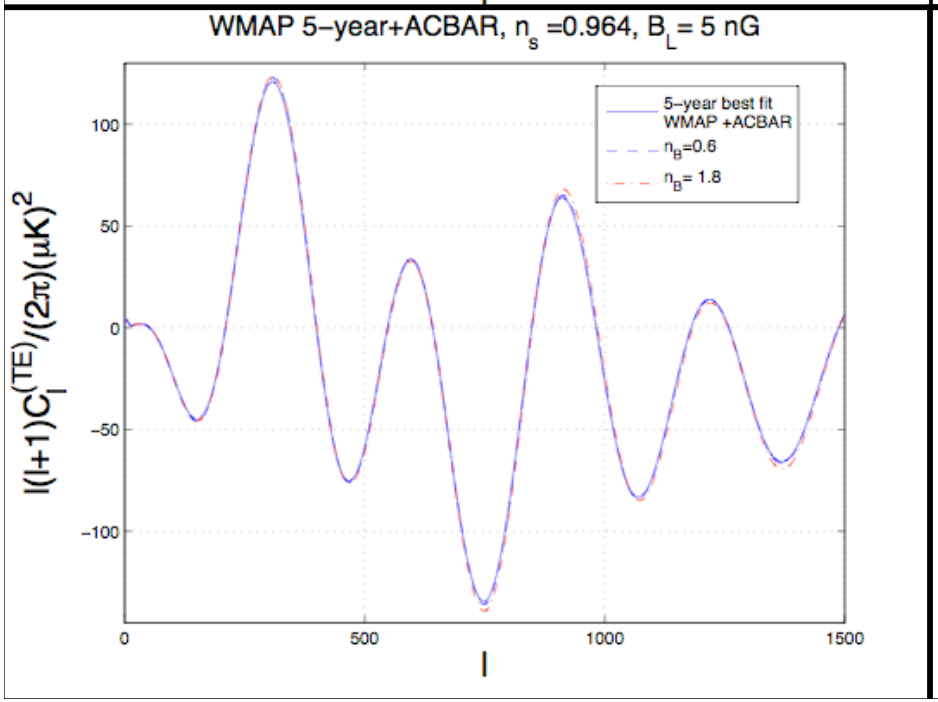


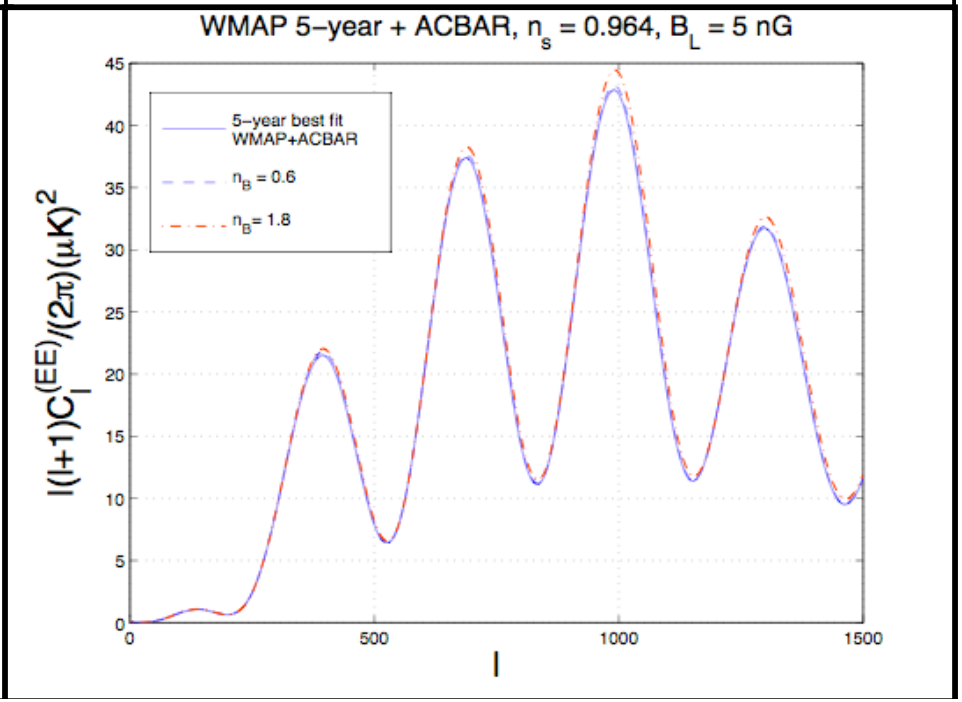
Magnetized TE and EE correlations/1

M. G. PRD 103007 (2009).

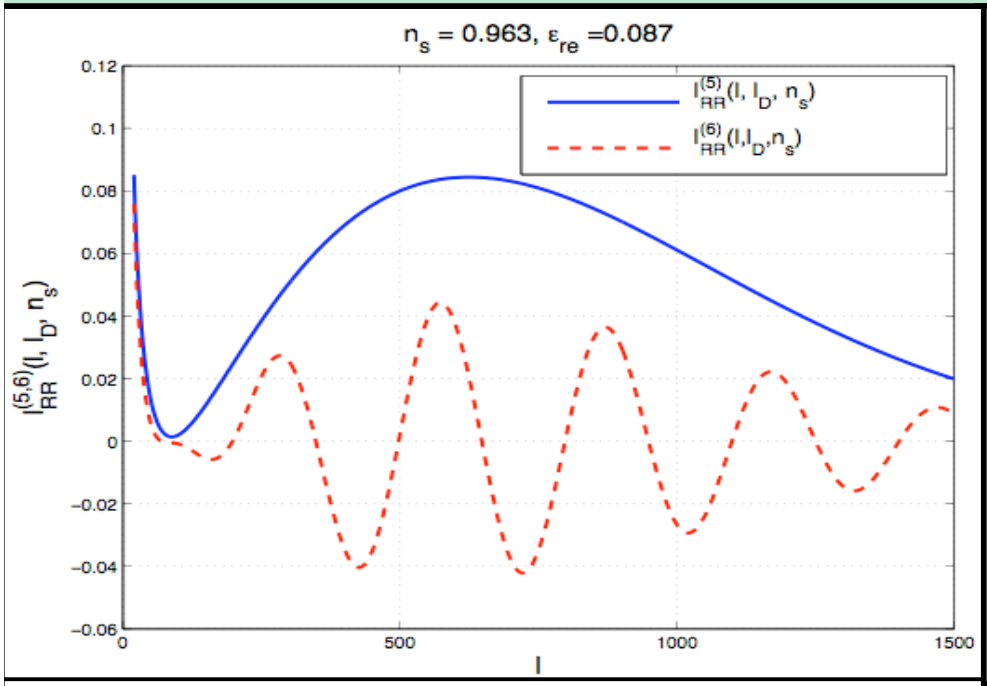


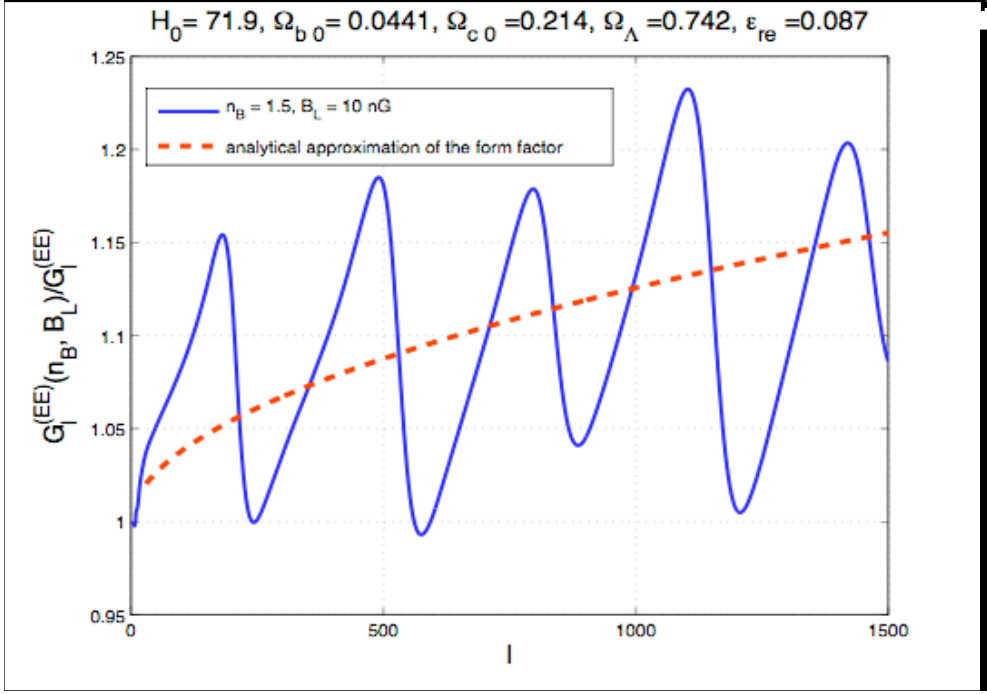






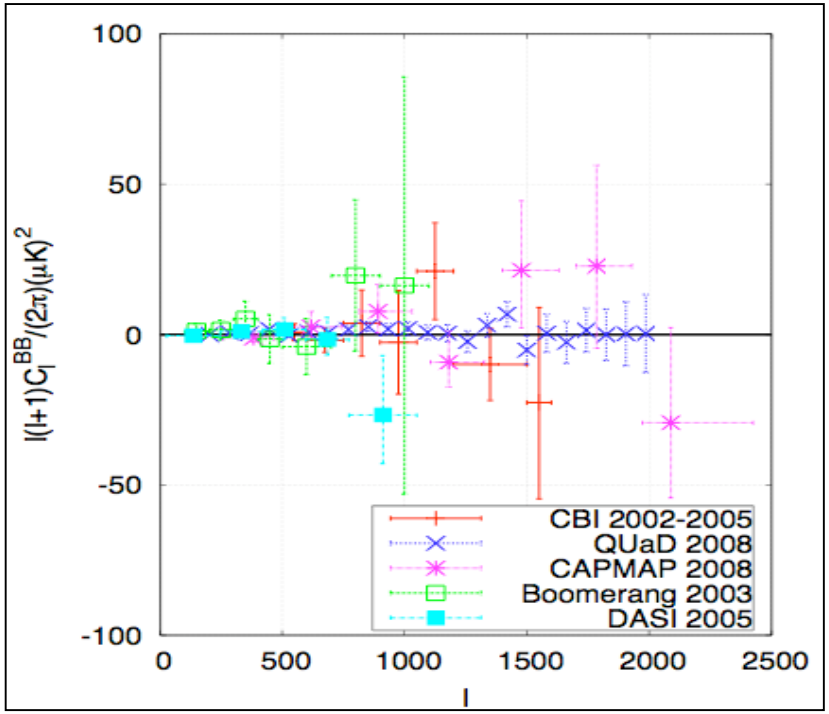
Magnetized TE and EE correlations/2

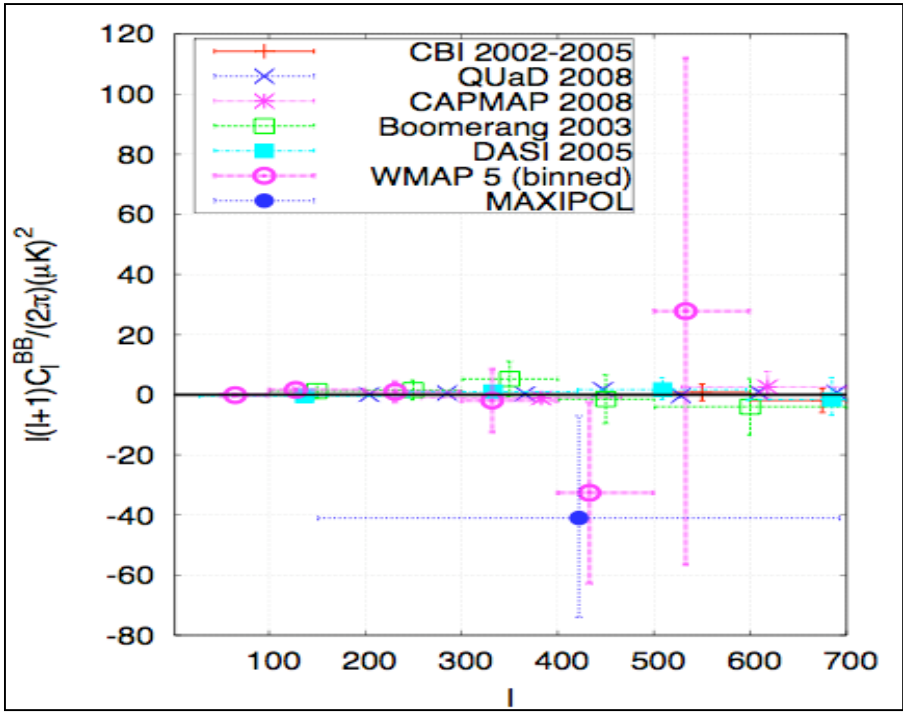




BB correlations/2

PRD 063007 (2009);





Experiments	Years	Data	Pivot frequencies	Upper limits on BB
Dasi	2000-2003	EE, TE, BB	26–36 GHz	$\mathcal{G}_\ell^{(\mathrm{BB})} \leq 50\,(\mu\mathrm{K})^2$
Boomerang	2003	EE, EB, BB	145 GHz	$\mathcal{G}_\ell^{(\mathrm{BB})} \leq 8.6\,(\mu\mathrm{K})^2$
Maxipol	2003	EE, EB, BB	140 GHz	$\mathcal{G}_\ell^{(\mathrm{BB})} \leq 112.3\,(\mu\mathrm{K})^2$
Quad	2007	EE,TE, TB	100/ 150 GHz	$\mathcal{G}_\ell^{(\mathrm{BB})} \leq 10\,(\mu\mathrm{K})^2$
Cbi	2002-2005	EE, TE, BB	26–36 GHz	$\mathcal{G}_\ell^{(\mathrm{BB})} \leq 3.76\,(\mu\mathrm{K})^2$
Capmap	2004-2005	EE, BB	35–46/84–100 GHz	$\mathcal{G}_\ell^{(\mathrm{BB})} \leq 4.8\,(\mu\mathrm{K})^2$
Wmap	2001-2006	EE, TE, BB	23–94 GHz	$\mathcal{G}_\ell^{(\mathrm{BB})} \leq 0.15\,(\mu\mathrm{K})^2$

BB correlations/3

$$C_\ell^{(\mathrm{BB})} = \sum_{j_1\ j_2} \mathcal{F}(\ell, j_1, j_2)\, C_{j_2}^{(\mathrm{EE})}(\ell_\mathrm{D}, \ell_\mathrm{t})\, C_{j_1}^{(\mathrm{FF})}(\ell_\mathrm{D}, \ell_\mathrm{t})$$

$$\to \int dj_1 \int dj_2 \mathcal{F}(\ell, j_1, j_2)\, C_{j_2}^{(\mathrm{EE})}(\ell_\mathrm{D}, \ell_\mathrm{t})\, C_{j_1}^{(\mathrm{FF})}(\ell_\mathrm{D}, \ell_\mathrm{t}),$$

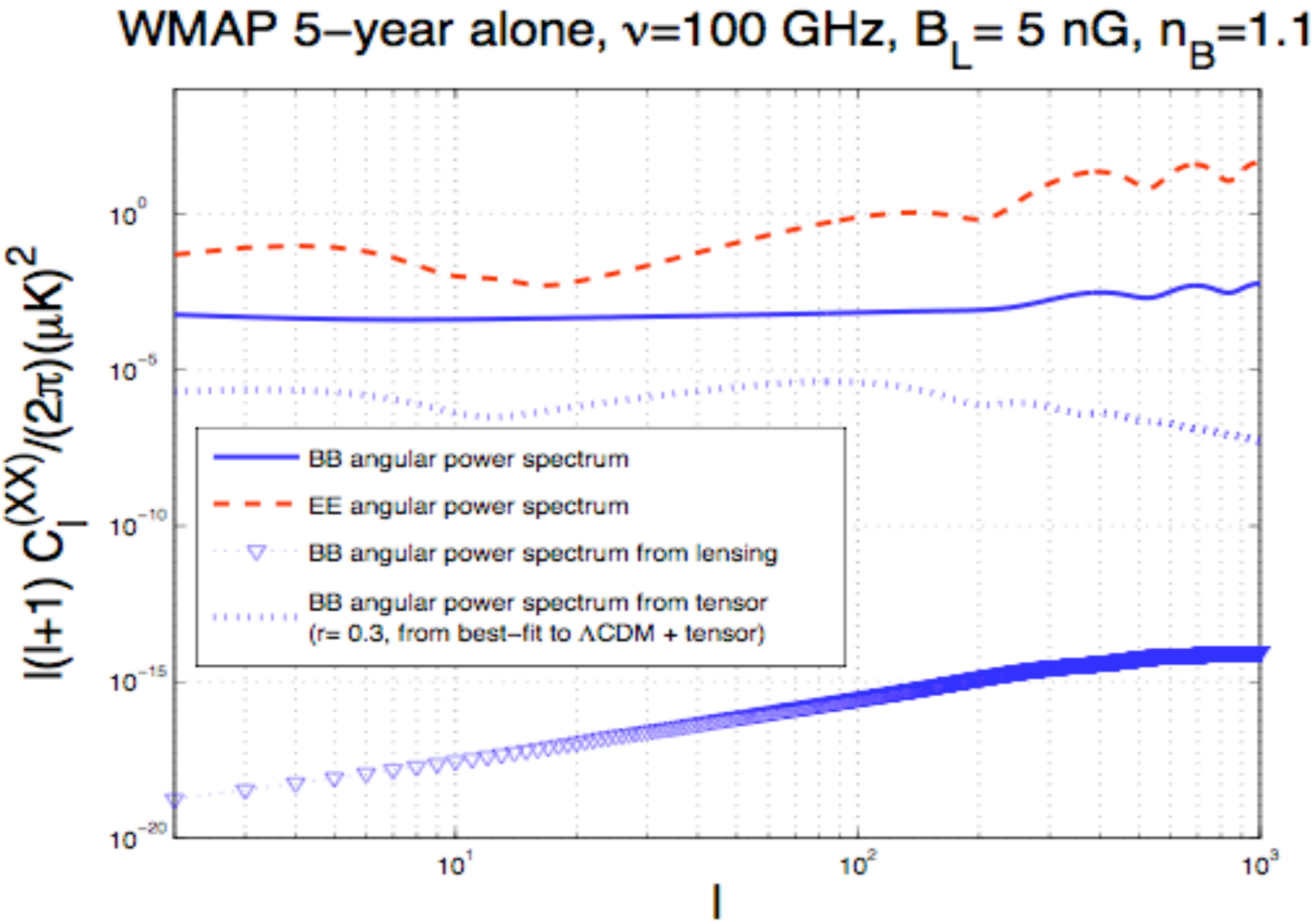
$$\mathcal{F}(\ell, j_1, j_2) = \frac{2j_2+1}{4\pi}\left(\frac{2j_1+1}{2\ell+1}\right)\frac{f(\ell, j_1, j_2)}{\ell\,(\ell-1)\,(\ell+1)\,(\ell+2)\,j_2\,(j_2-1)\,(j_2+1)\,(j_2+2)}\,|C_{j_1\ 0\ j_2\ 0}^{\ell\ 0}|^2,$$

$$\begin{aligned} f(\ell, j_1, j_2) &= 2j_1j_2(j_1+1)(j_2+1) + 2j_1\ell(j_1+1)(\ell+1) - \ell^2(\ell+1)^2 \\ &- j_1^2(j_1+1)^2 - j_2^2(j_2+1)^2 + 2j_2(j_2+1) + 2\ell(\ell+1) - 2j_1(j_1+1), \end{aligned}$$

$$|C_{j_1\ 0\ j_2\ 0}^{\ell\ 0}|^2 = \frac{2\ell+1}{\pi}\,\frac{1}{\sqrt{4j_1^2j_2^2 - (\ell^2 - j_1^2 - j_2^2)^2}}.$$

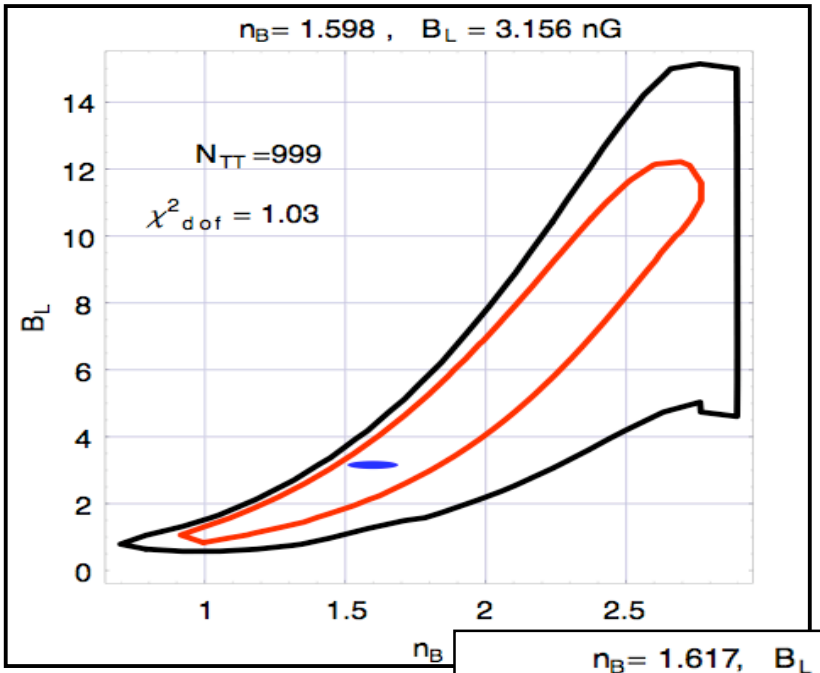
Semiclassical expression of Clebsch-Gordon coefficients (Ponzano & Regge 1968)
Inonu-Wigner contraction
Projective techniques ? (Regge symbols & magic squares?)

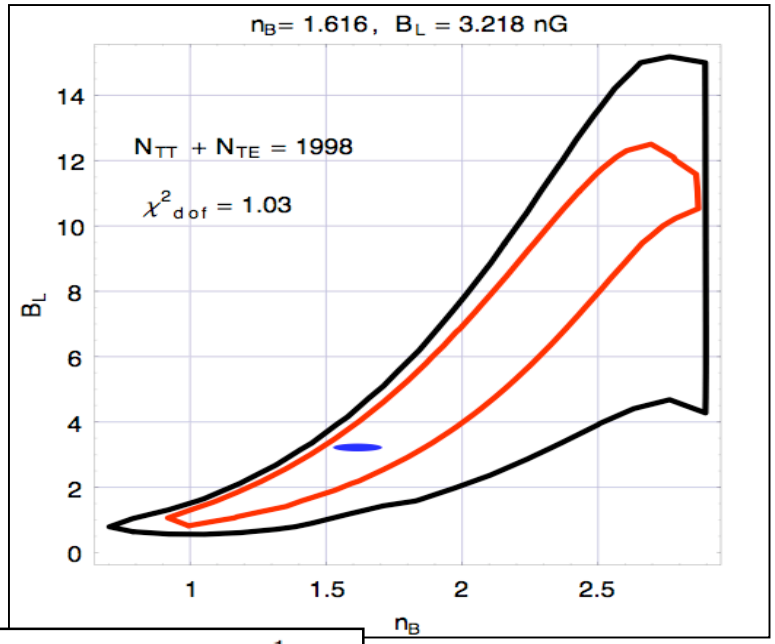
BB correlations/4

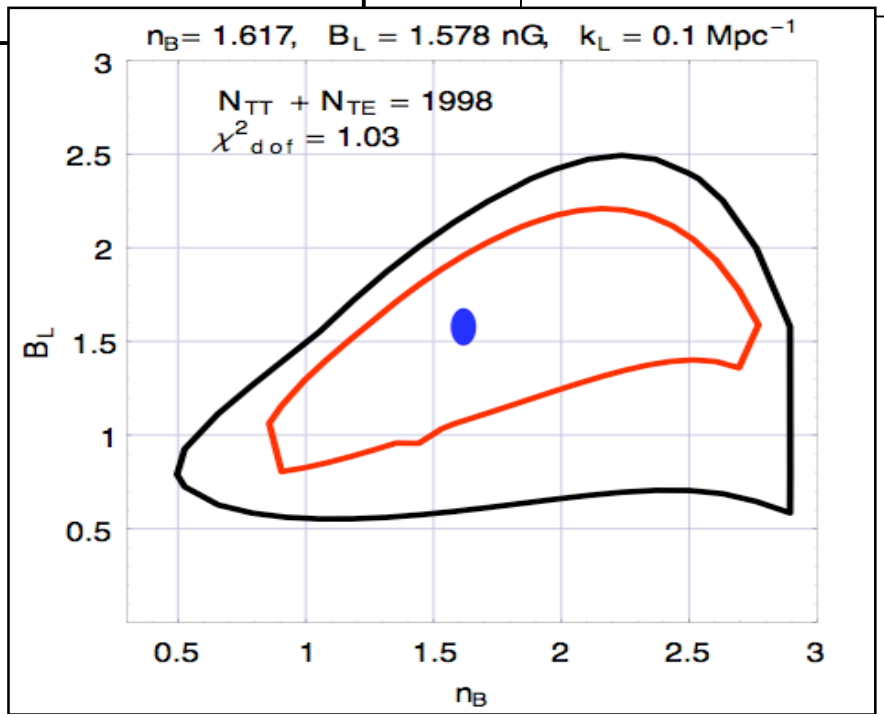


Parameter space/1

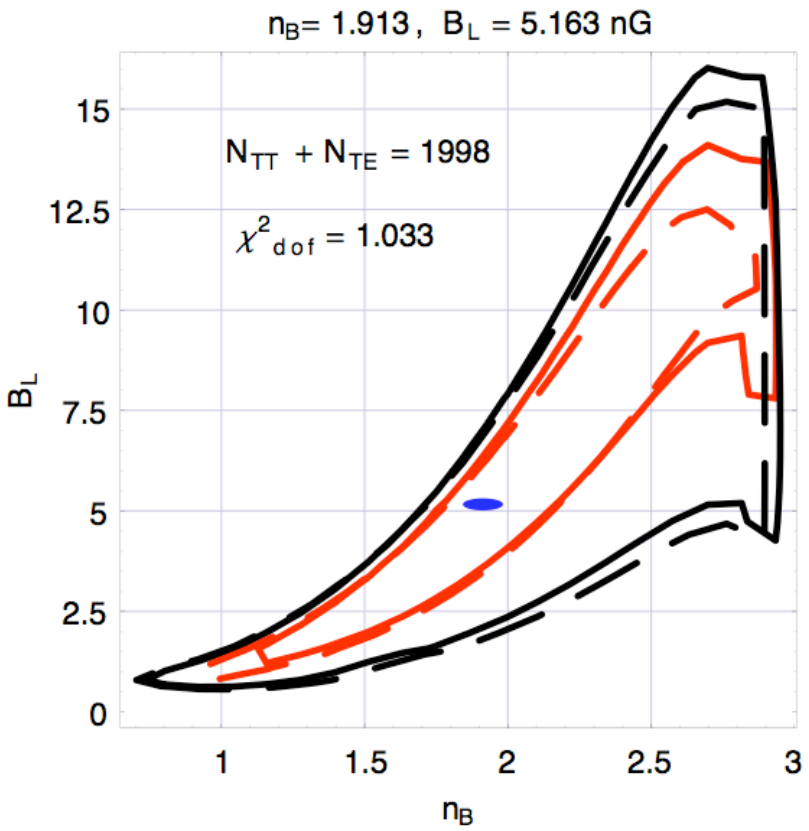
PRD 79 121302 (2009);

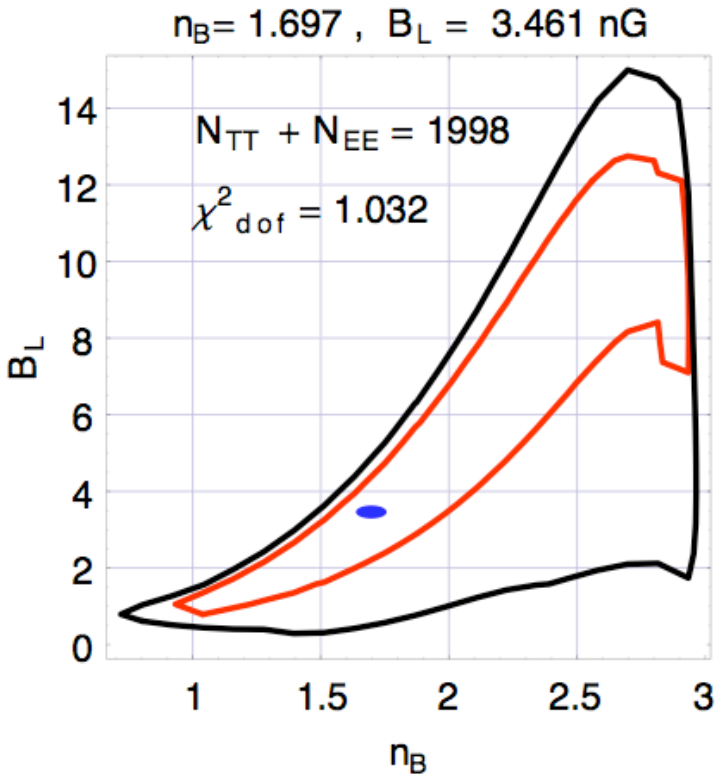






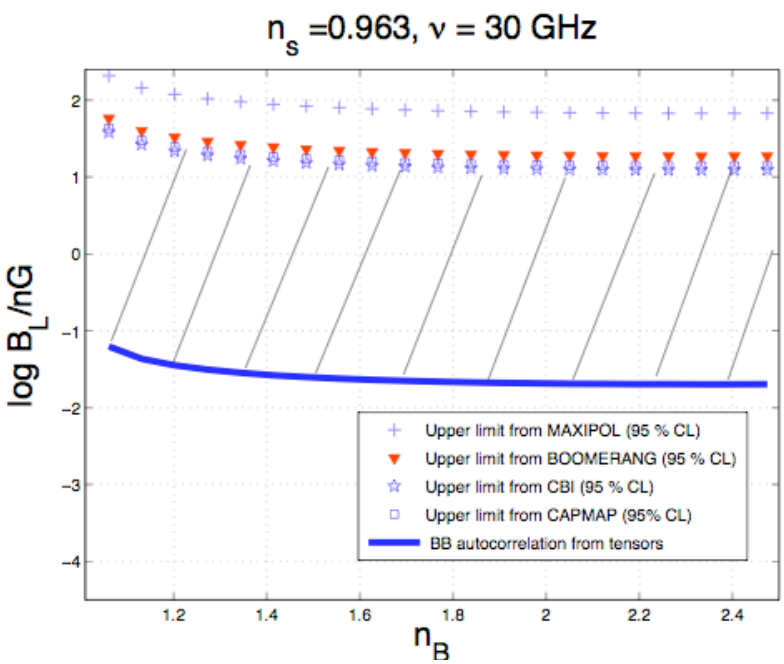
Parameter space/2

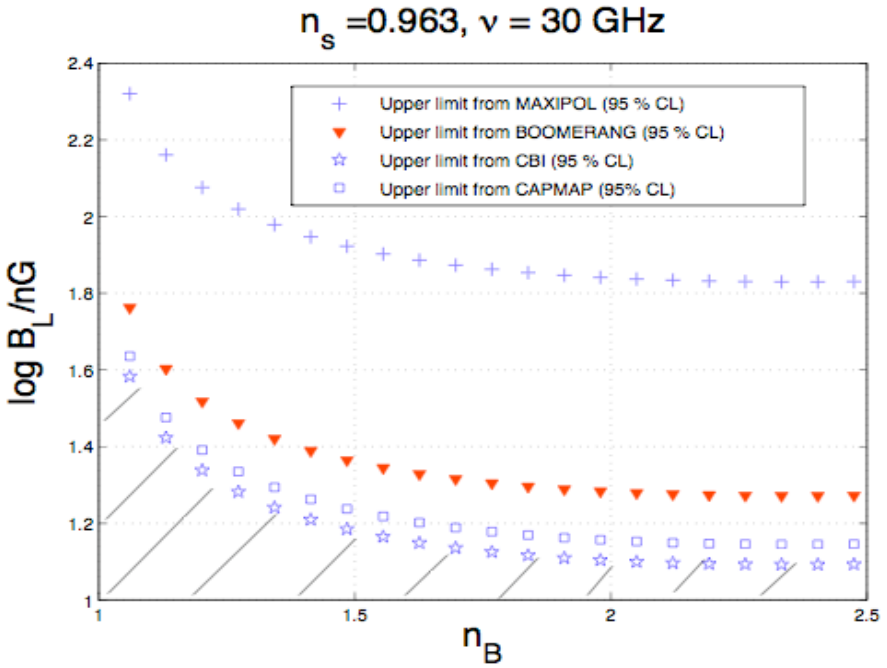


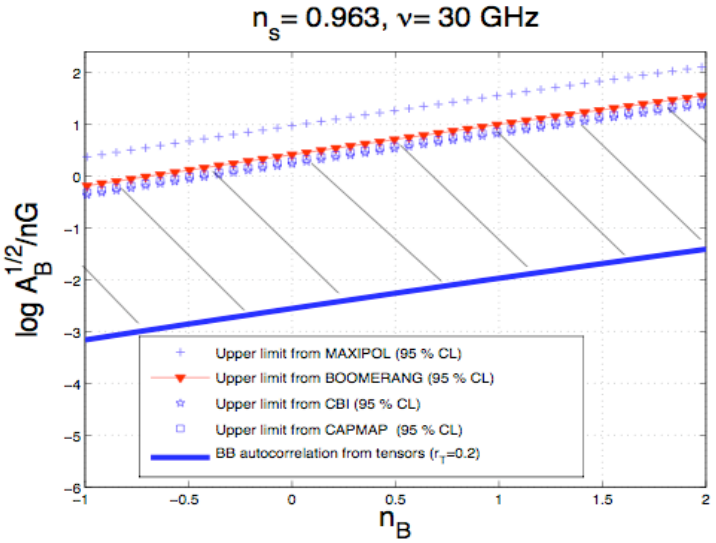


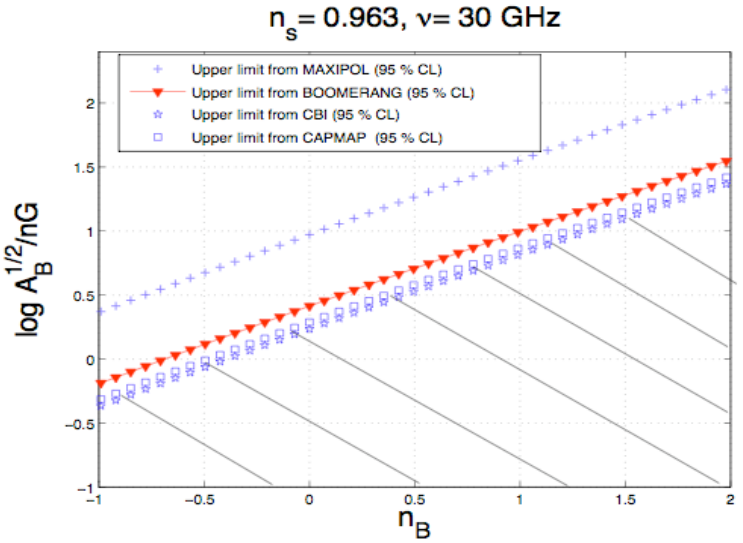
$$(n_{\rm B}, B_{\rm L})_{\Lambda{\rm CDM}} = (1.598,\, 3.156{\rm nG}) \rightarrow (n_{\rm B}, B_{\rm L})_{w{\rm CDM}} = (1.883,\, 4.982\,{\rm nG}),$$
$$(n_{\rm B}, B_{\rm L})_{\Lambda{\rm CDM}} = (1.616,\, 3.218{\rm nG}) \rightarrow (n_{\rm B}, B_{\rm L})_{w{\rm CDM}} = (1.913,\, 5.163\,{\rm nG}),$$

Parameter space/3



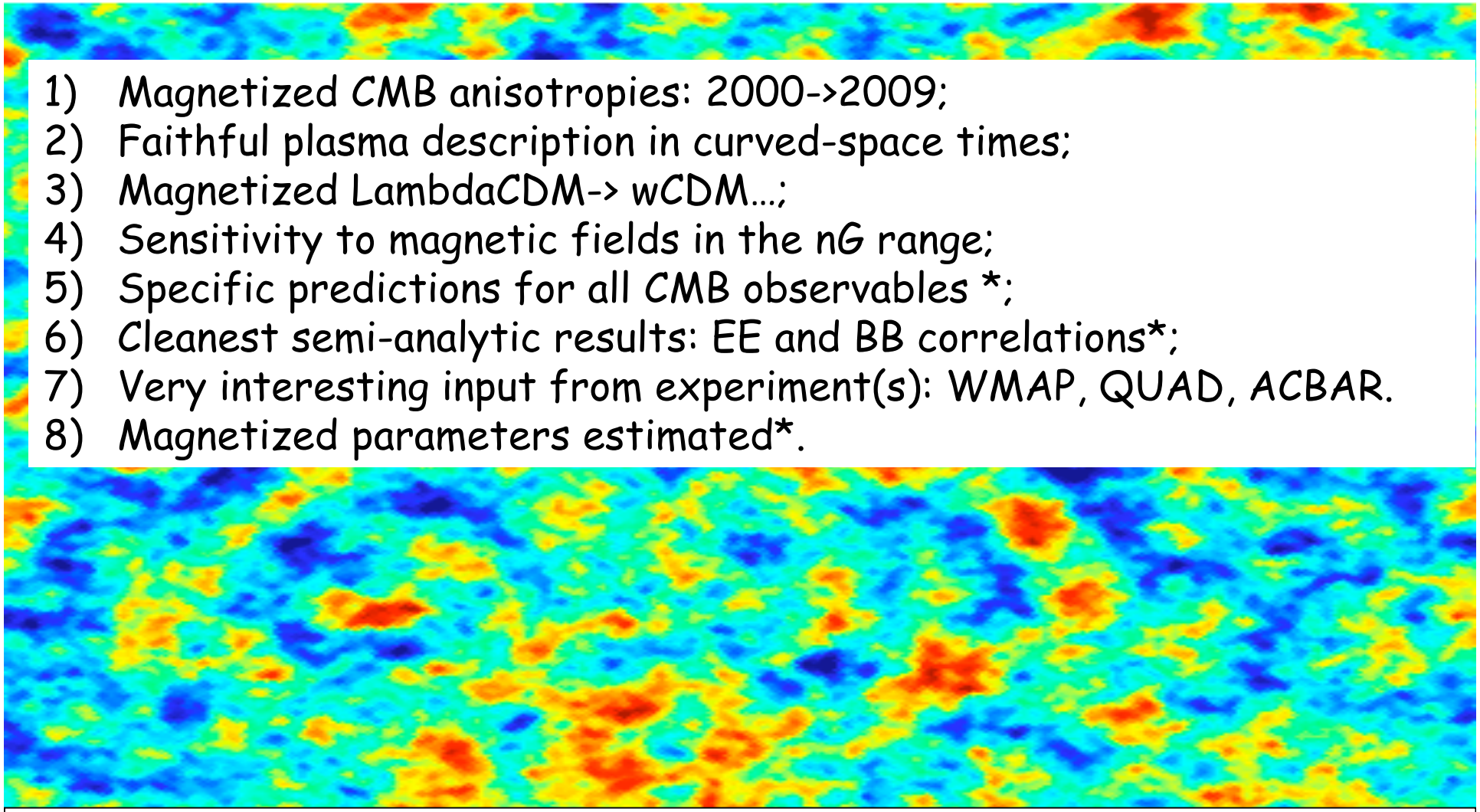






Summary

WMAP 5 + B=5nG, nB=1.5, theta=5', 20 x 20 (deg)^2



1) Magnetized CMB anisotropies: 2000->2009;
2) Faithful plasma description in curved-space times;
3) Magnetized LambdaCDM-> wCDM…;
4) Sensitivity to magnetic fields in the nG range;
5) Specific predictions for all CMB observables *;
6) Cleanest semi-analytic results: EE and BB correlations*;
7) Very interesting input from experiment(s): WMAP, QUAD, ACBAR.
8) Magnetized parameters estimated*.

* First time results