Surprises from Second-Order Cosmological Perturbation Theory

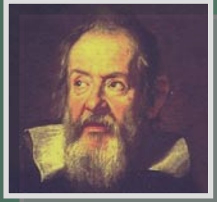
Sabino Matarrese

Dipartimento di Fisica *G. Galilei*

Universita' degli Studi di Padova

ITALY

email: matarrese@pd.infn.it



Contents

- *Generalities on GR 2-nd order cosmological PT*
- *Primordial non-Gaussian perturbations and observational constraints*
- *B-mode CMB polarization from second-order vector and tensor modes*
- *Seed magnetic field generation from density fluctuations*
- *Back-reaction: does acceleration require dark energy?*

First-order metric perturbations in the Poisson gauge (*dust case*)

$$ds^2 = a^2(\tau)\left[-(1+2\phi)d\tau^2 - 2V_i d\tau dx^i + ((1-2\psi)\delta_{ij} + h_{ij})dx^i dx^j\right]$$

$$\nabla^2\nabla^2(\phi-\psi) = 0$$ ← scalar modes

$$\nabla^2\nabla^2 V_i = 0$$ ← vector modes

$$V^i_{,i} = 0$$

tensor modes →

$$\nabla^2\nabla^2(\ddot{h}_{ij} + H\dot{h}_{ij} - \nabla^2 h_{ij}) = 0$$

$$h_{ij} = h_{ji} \qquad h^i_i = h^i_{j,i} = 0$$

Second-order metric perturbations in the Poisson gauge (*dust case*)

$$ds^2 = a^2(\tau)[-(1+2\phi)d\tau^2 - 2V_i d\tau dx^i + ((1-2\psi)\delta_{ij} + h_{ij})dx^i dx^j]$$

$$\nabla^2\nabla^2(\phi-\psi) = -2\nabla^2\nabla^2\varphi^2 - \frac{1}{2}\nabla^2(2\partial^l\varphi\partial_l\varphi + 3H^2 v^2)$$

scalar modes

$$+ \frac{3}{2}\partial^i\partial_j(2\partial_i\varphi\partial^j\varphi + 3H^2 v_i v^j)$$

$$\nabla^2\nabla^2 V_i = 16\pi G a^2\partial^j(v_j\partial_i\rho - v_i\partial_j\rho)$$

$$\nabla^2\nabla^2(\ddot{h}_{ij} + H\dot{h}_{ij} - \nabla^2 h_{ij}) = 2[\nabla^2\partial^k\partial_l R^l_k\delta^i_j$$

vector modes

$$+ 2\nabla^2(\nabla^2 R^i_j - \partial^k\partial_j R^i_k - \partial^i\partial_l R^l_j)$$

tensor modes

$$+ \partial^i\partial_j\partial^k\partial_l R^l_k]$$

$$R^i_j \equiv \partial^i\varphi\partial_j\varphi - \frac{1}{2}(\nabla\varphi)^2\delta^i_j + 4\pi G a^2\rho\left(v^i v_j - \frac{1}{3}v^2\delta^i_j\right)$$

Synchronous and comoving gauge: Tomita 1971; Matarrese, Pantano & Saez 1994;
Poisson gauge: Matarrese, Mollerach & Bruni 1998
Extended to fully non-linear case by Carbone & Matarrese (2004)

Non-Gaussian primordial perturbations from Inflation

Viviana Acquaviva (SISSA, Trieste)
Nicola Bartolo (Sussex → ICTP, Trieste)
Michele Liguori (Phys. Dept., Padova)
Sabino Matarrese (Phys. Dept., Padova)
Antonio Riotto (INFN, Padova)
...

based on ...

review on
non-Gaussianity
from Inflation

- Acquaviva V., Bartolo N., Matarrese S. & Riotto A. 2003, Nucl. Phys. B **667** 119
- Bartolo N., Komatsu E., Matarrese S. & Riotto A. 2004, Phys. Rept. **402** 103 (astro-ph/0406398)
- Bartolo N., Matarrese S. & Riotto A. 2002, Phys. Rev. D 65 103505
- Bartolo N., Matarrese S. & Riotto A. 2004, Phys. Rev. D **69** 043503
- Bartolo N., Matarrese S. & Riotto A. 2004, JCAP **01** 003
- Bartolo N., Matarrese S. & Riotto A. 2004, JHEP **04** 006
- Bartolo N., Matarrese S. & Riotto A., 2004, Phys. Rev. Lett., in press (astro-ph/0407505)
- Liguori M., Matarrese S. & Moscardini L. 2003, ApJ **597** 56

Why (non-) Gaussian?

Gaussian ⟷ free (i.e. non-interacting) field

- collection of independent harmonic oscillators (no mode-mode coupling)
- the motivation for Gaussian initial conditions (the standard assumption) ranges from mere simplicity to the use of the Central Limit Theorem, to the property of inflation produced seeds (… see below)

large-scale phase coherence ⟷ non-linear gravitational dynamics

The present-day view on non-Gaussianity

- Alternative structure formation models of the eighties considered strongly non-Gaussian primordial fluctuations.
- The increased accuracy in CMB and LSS observations has, however, excluded this extreme possibility.
- The present-day challenge is to either detect or constrain mild or even weak deviations from primordial Gaussian seeds.
- Deviations of this type are not only possible but are unavoidably predicted in the standard perturbation generating mechanism provided by inflation.

“Non-Gaussian = non-dog”

S. F. Shandarin

- Need a model able to parametrize deviations from the Gaussian behaviour in a cosmological framework

- A simple class of **mildly** non-Gaussian perturbations is described by a sort of Taylor expansion around the Gaussian case:

$$\Phi = \phi + f_{NL}\,\phi^2 + g_{NL}\,\phi^3 + \ldots \text{ const.}$$

where Φ is the peculiar gravitational potential, ϕ is a Gaussian field, f_{NL}, g_{N}, etc. … are dimensionless parameters quantifying the non-Gaussian (i.e. non-linear) strength

Non-Gaussianity and scale-invariance

Following *Otto, Politzer, Preskill & Wise* (1986) and *Grinstein & Wise* (1986), if the scales of the perturbation-generating process are negligible w.r.t. the cosmologically relevant scales, a *generalized scale-invariance* criterion applies:

given the linear density fluctuation field

$\delta(\underline{k},\tau) = \varepsilon(\underline{k})\, D_+(\tau)$,

with $D_+(\tau)$ the linear growing mode and τ the conformal time, scale-invariance requires

$$< \varepsilon(\lambda\underline{k}_1)\, \varepsilon(\lambda\underline{k}_2) \ldots \varepsilon(\lambda\underline{k}_n)>_{connected} = \lambda^{-n} <\varepsilon(\underline{k}_1)\, \varepsilon(\underline{k}_2)\, \varepsilon(\underline{k}_n)>_{connected}$$

which extends to arbitrary order the scaling implicit in the *Harrison-Zel'dovich* power-spectrum

$$< \varepsilon(\lambda\underline{k}_1)\, \varepsilon(\lambda\underline{k}_2) > \sim k_1\, \delta^{(3)}(\underline{k}_1 + \underline{k}_2)$$

The non-Gaussian model

- Many primordial (inflationary) models of non-Gaussianity can be represented in configuration space by the general formula (e.g. *Verde et al. 2000; Komatsu & Spergel 2001*)

$$\Phi = \phi_L + f_{NL} * (\phi_L^2 - \langle\phi_L^2\rangle)$$

where Φ is the large-scale gravitational potential, ϕ_L its linear Gaussian contribution and f_{NL} is the dimensionless *non-linearity parameter* (or more generally *non-linearity function*). The percent of non-Gaussianity in CMB data implied by this model is

$$\text{NG \%} \sim 10^{-5} \, |f_{NL}|$$

$< 10^{-3}$ from *WMAP*

Classify Inflationary Models

- The shape of the inflaton potential $V(\varphi)$ determines the observables.
- It is standard practice to use three "slow-roll" parameters to characterize it:

slow-roll conditions

ε "slope" of the potential ~ $(V'/V)^2$ ⑥ 1

η "curvature" of the potential ~V''/V ~ ε ⑥ 1

ξ "jerk" of the potential ~$(V'/V)(V'''/V)$ ~ ε^2

Classify Inflationary Models

Ratio of tensor to scalar modes

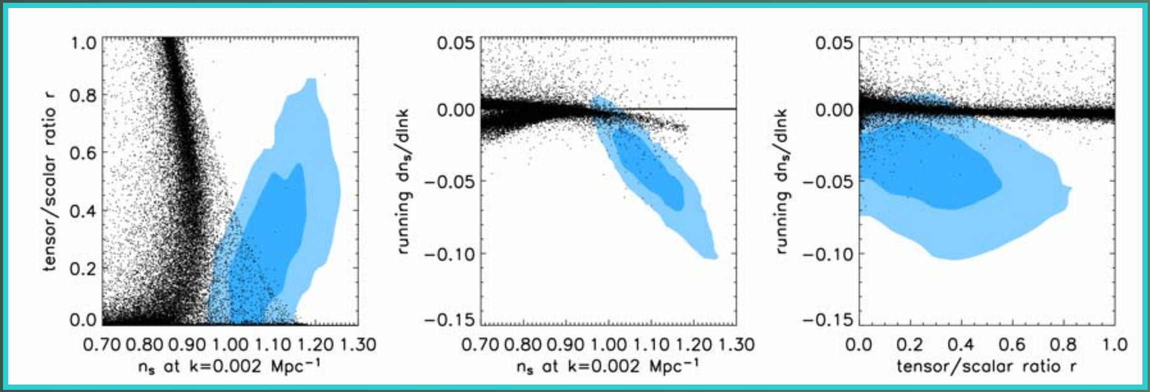
Tilt of scalar perturbations

Running scalar spectral index

$$\begin{cases} r = 16\varepsilon \\ n_s = 1 - 6\varepsilon + 2\eta \\ dn_s / d\ln k = -2\xi + 16\varepsilon\eta - 24\varepsilon^2 \end{cases}$$

Each point is a "viable" slow-roll model, able to sustain inflation for sufficient *e*-foldings to solve the horizon problem and make the Universe (nearly) flat.

Monte Carlo simulations following (*Kinney 2002*) flow-equation technique



Peiris et al. 2003

Where does large-scale non-Gaussianity come from?

- ✓ *Falk et al. (1993)* found $f_{\mathrm{NL}} \sim \xi \sim \varepsilon^2$ (from non-linearity in the inflaton potential in a fixed de Sitter space) in the standard single-field slow-roll scenario

- ✓ *Gangui et al. (1994),* using stochastic inflation found $f_{\mathrm{NL}} \sim \varepsilon$ (from second-order gravitational corrections during inflation). *Acquaviva et al. (2003)* and *Maldacena (2003)* confirmed this estimate (up to numerical factors and momentum-dependent terms) with a full second-order approach

- ✓ *Bartolo et al. (2004)* show that second-order corrections <u>after inflation</u> enhance the primordial signal leading to $f_{\mathrm{NL}} \sim 1$

Non-Gaussianity requires more than linear theory ...

The leading contribution to higher-order statistics (such as the bispectrum, i.e. the FT of the three-point function) comes from second-order metric perturbations around the RW background (unless the primordial non-Gaussianity is very strong)

"… the linear perturbations are so surprisingly simple that a perturbation analysis accurate to second order may be feasible …"

Sachs & Wolfe 1967

Evaluating non-Gaussianity from inflation: summary

- Evaluate non-Gaussianity during inflation by a self-consistent second-order calculation.
- Evolve scalar (vector and tensor) perturbations to second order after inflation outside the horizon, matching a conserved second-order gauge-invariant variable, such as the comoving curvature perturbation $\zeta^{(2)}$ defined by *Malik & Wands (2004)*, or the similar quantity defined by *Salopek & Bond (1990)*, $\zeta^{(2)}_{SB} = \zeta^{(2)} - 2\,(\zeta^{(1)})^2$, to its value at the end of inflation (accurately accounting for the Universe reheating after inflation).
- Evolve them consistently inside the horizon → this should involve a calculation of the radiation transfer function to second order!
- On large scales, account for: ***i)*** Sachs-Wolfe-like second-order temperature fluctuations (*Mollerach & Matarrese 1997*), ***ii)*** second-order intrinsic temperature anisotropies at last-scattering.

Non-Gaussianity from Inflation: results

The amount of non-Gaussianity from a wide class of models, including single-field slow-roll inflation, curvaton (*Mollerach 1990; Moroi & Takahashi 2001; Enqvist & Sloth 2002; Lyth & Wands 2002*) and modulated reheating (*Hamazaki & Kodama 1996; Dvali et al. 2003; Zaldarriaga 2003; Kofman 2003*), follows a universal (second-order) relation:

$$\Delta T/T = 1/3\,(\phi_L + \phi_{NL})$$

$$\phi_{NL} = f_{NL} * \phi_L^2 + \mathit{const.}$$

$$f_{NL} = f_{NL}^0 - 3(k_1^4 + k_2^4)/2k^4 + (\underline{k}_1 \cdot \underline{k}_2 / k^2) \cdot [4 - 3\,(\underline{k}_1 \cdot \underline{k}_2 / k^2)], \qquad \underline{k} = \underline{k}_1 + \underline{k}_2$$

Sachs-Wolfe limit; replaced by full transfer function in CMB maps

Inflation models and f_{NL}

model	$f_{NL}(\mathbf{k}_1,\mathbf{k}_2)$	comments
single-field inflation	$7/3 - g(\mathbf{k}_1, \mathbf{k}_2)$	$g(k_1, k_2)=3(k_1^4+k_2^4)/2k^4+(\underline{k}_1 \cdot \underline{k}_2) \cdot [4-3(\underline{k}_1 \cdot \underline{k}_2)/k^2]/k^2$, $\underline{k}=\underline{k}_1+\underline{k}_2$
curvaton scenario	$2/3 - 5r/6 + 5/4r - g(\mathbf{k}_1, \mathbf{k}_2)$	$r \sim (\rho_\sigma/\rho)_{decay}$
modulated reheating	$13/12 - I - g(\mathbf{k}_1, \mathbf{k}_2)$	$I = -5/2 + 5\Gamma / (12\,\alpha\Gamma_1)$ $I = 0$ (*minimal case*)
multi-field inflation	up to 10^2	*order of magnitude estimate of the absolute value*
"unconventional" inflation set-ups		
warm inflation	typically 10^{-1}	*second-order corrections not included*
ghost inflation	$-140\,\beta\,\alpha^{-3/5}$	*post-inflation corrections not included*
D-cceleration	$-0.1\,\gamma^2$	*post-inflation corrections not included*

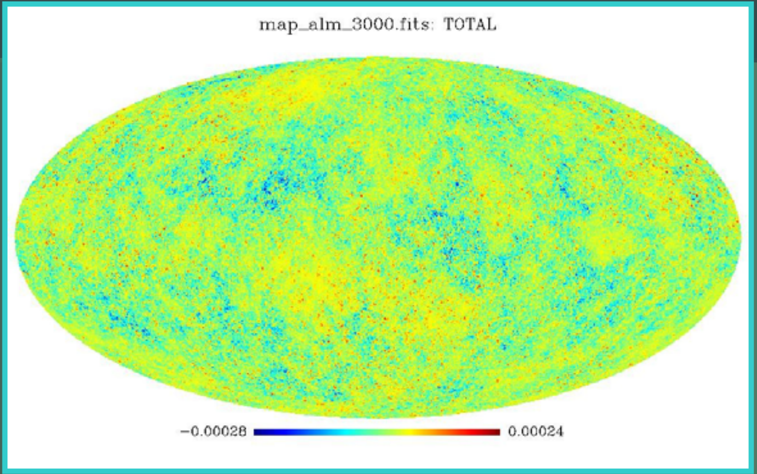
Non-Gaussian CMB maps: Planck resolution

Liguori, Matarrese & Moscardini 2003, ApJ **597**, 56

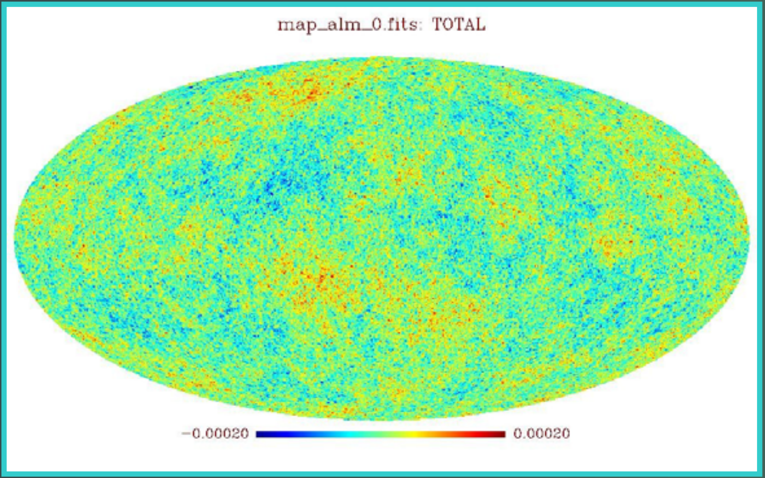
5' resolution

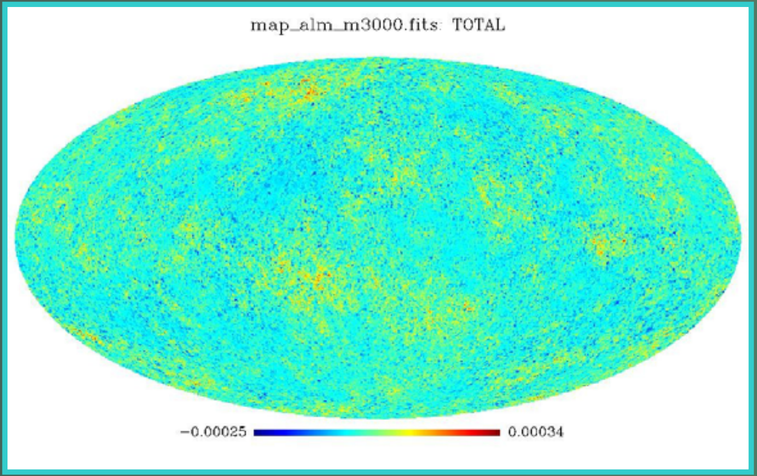
l_{max} = 3000, N_{side}=2048

f_{NL} = 3000



f_{NL} = 0

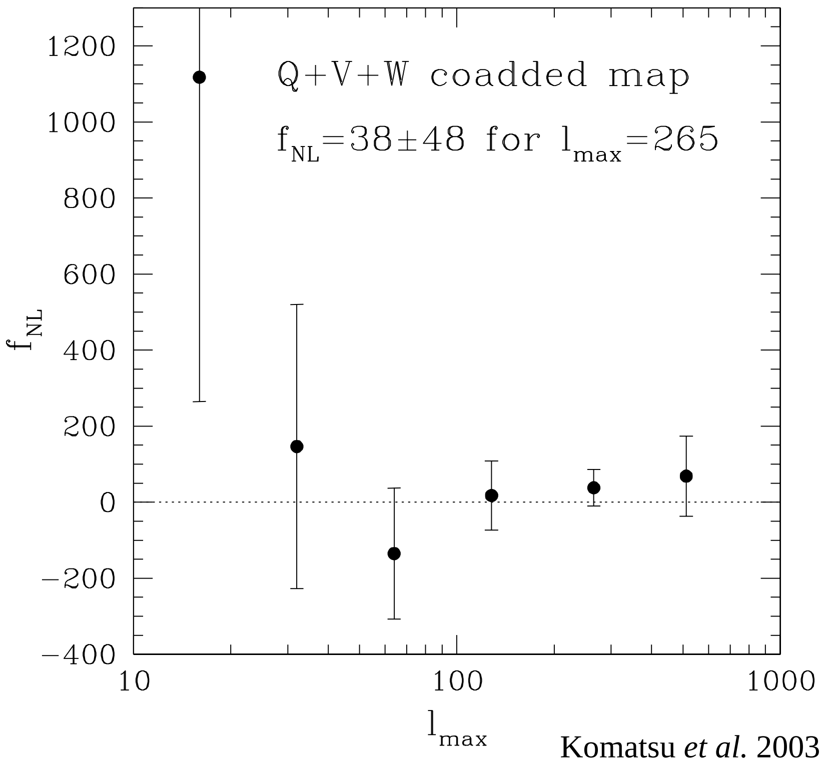




f_{NL} = - 3000

Observational constraints on f_{NL}

- The strongest limits on non-Gaussianity so far come from WMAP data. *Komatsu et al. (2003)* find (at 95% cl). $-58 < f_{NL} < 134$
- According to *Komatsu & Spergel (2001)* using the angular bispectrum one can reach values as low as $|f_{NL}| = 20$ with *WMAP* & $|f_{NL}| = 5$ with *Planck* can be achieved.
- The role of the f_{NL} momentum-dependent part is being explored (Komatsu, *Liguori, Matarrese & Riotto, in prep.*) as a characteristic inflation signature reflecting into some specific triangle configuration of the bispectrum.



Komatsu *et al.* 2003

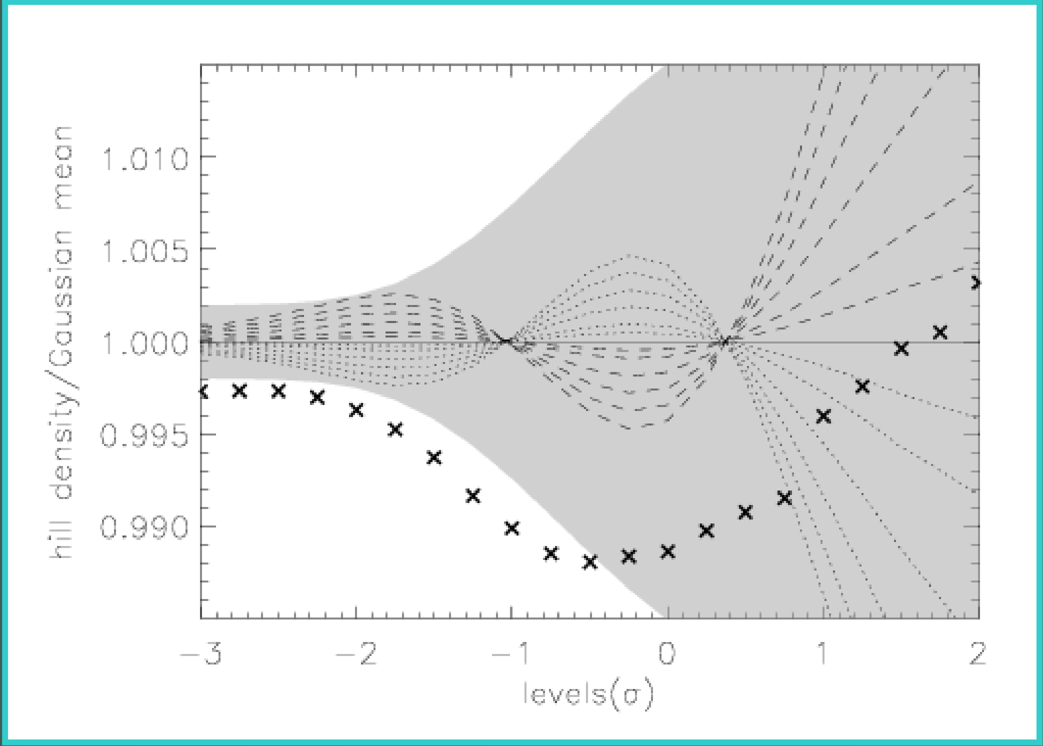
Statistical analysis of NG CMB maps vs. WMAP

Local curvature of CMB anisotropies

Density of hills (where the Hessian eigenvalues are both positive) as a function of the threshold ν, for different values of f_{NL}. The grey band is the 1σ confidence level. The solid crosses are the *WMAP* data. We find

$$f_{NL} = -5 \pm 175$$

at the 2σ level



Cabella P., Liguori M., Hansen F., Marinucci D., Matarrese S., Moscardini L. & Vittorio N. 2004

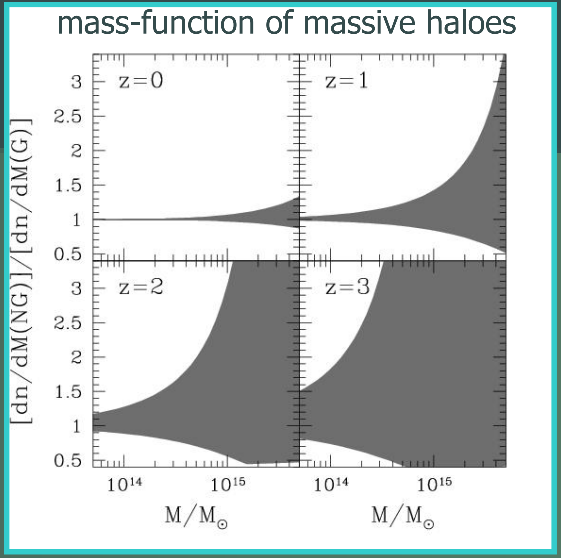
Searching for non-Gaussianity with rare events

- Besides using standard statistical estimators, like bispectrum, trispectrum, three and four-point function, skewness , etc. …, one can look at the tails of the distribution, i.e. at rare events.

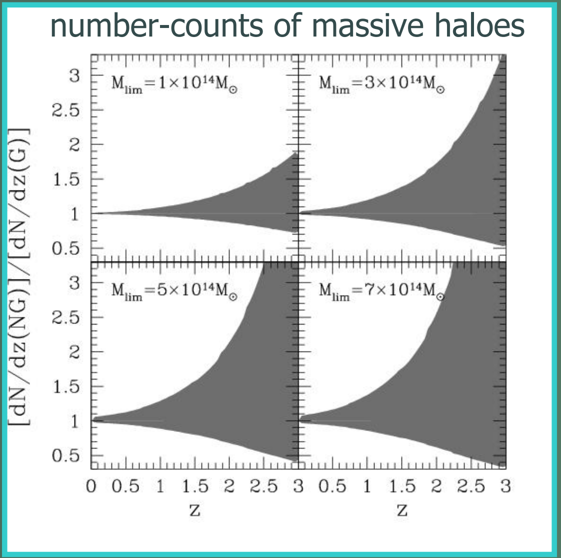
- Rare events have the advantage that they often maximize deviations from what predicted by a Gaussian distribution, but have the obvious disadvantage of being … rare!

- *Matarrese, Verde & Jimenez (2000)* and *Verde, Jimenez, Kamionkowski & Matarrese (2001)* have shown that clusters at high redshift (z>1) can probe NG down to $f_{NL} \sim 10^2$ which is, however, not competitive with future CMB (Planck) constraints.

- Primordial non-Gaussianity would also strongly affect the abundance of the first non-linear objects in the Universe, thereby affecting the reionization epoch *(Chen et al. 2003)* ➔ is the WMAP result $z_{reion} \sim 20$-30 more likely?



Komatsu et al. 2003



Searching for non-Gaussianity with LSS

- *Verde et al. (1999, 2001)* showed that constraints on primordial non-Gaussianity in the gravitational potential from "local" large redshift-surveys like 2dF and SDSS are not competitive with CMB ones: f_{NL} has to be larger than $10^2 – 10^3$ in order to be detected as a sort of non-linear bias in the galaxy-to-dark matter density relation. However LSS gives complementary constraints, as it probes NG on different scales than CMB. Weak lensing statistics might also provide interesting constraints *(Lesgourgues, Liguori, Matarrese & Riotto 2004).*

- Going to redshift z~1 helps (but one would surveys covering a large fraction of the sky). *Scoccimarro et al. (2004)* claim that a bispectrum analysis over a hypothetical all-sky survey at z~1 might detect NG down to $f_{NL} \sim 1$ (quite likely too optimistic because of the large wave-number range used in the estimate and wide survey area assumed).

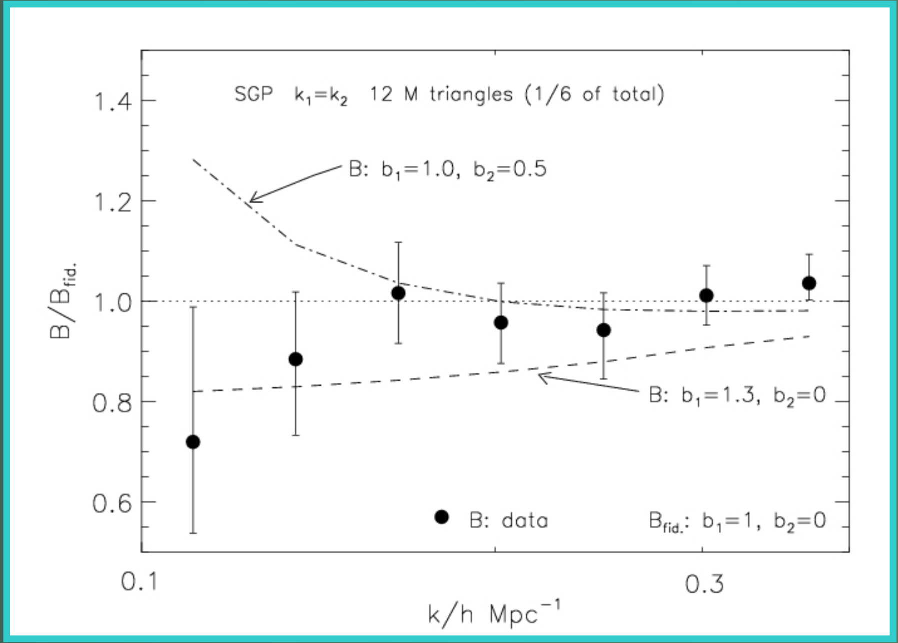
- A promising new technique could be provided by 21-cm background anisotropies *(Cooray 2004),* as the "effective" NG strength in the underlying CDM overdensity increases with redshift (Porciani & Matarrese, in prep.).

- The non-Gaussianity level perceived in the density field is not merely that arising from a quadratic term in the gravitational potential ($\delta \sim \nabla^2 \varphi^2$): a non-trivial and model-independent configuration dependence always arises, which might help NG detection in LSS analyses *(Bartolo, Matarrese & Riotto, in prep.).*

Bias and Ω_0 from the 2dFGRS

✓Analysing the bispectrum of 2dF galaxies *Verde, Heavens, Percival, Matarrese & 2dF team (2002)* found

$$b = 1.04 \pm 0.11$$
$$b_2 = -0.05 \pm 0.08$$



b_2 is where non-Gaussianity may be hidden!!

Conclusions on NG

- Contrary to earlier naive expectations, some level of non-Gaussianity is generically present in all inflation models
- The level of non-Gaussianity predicted in the simplest inflation models is slightly below the minimum value detectable by *Planck*, but the predicted angular dependence of f_{NL} *(Komatsu, Liguori, Matarrese & Riotto, in prep.),* extensive use of simulated non-Gaussian CMB maps, measurements of polarization and use of alternative statistical estimators might help non-Gaussianity detection down to $f_{NL} \sim 1$. Alternative techniques based on clustering at high redshift might also become viable in the future.
- Constraining or detecting non-Gaussianity will become a powerful tool to discriminate among competing scenarios for perturbation generation (*standard slow-roll inflation, curvaton and modulated-reheating scenarios, multi-field or ghost inflation, …*) some of which imply large non-Gaussianity
- Predicting or constraining non-Gaussianity should be considered as a branch of *Precision Cosmology*

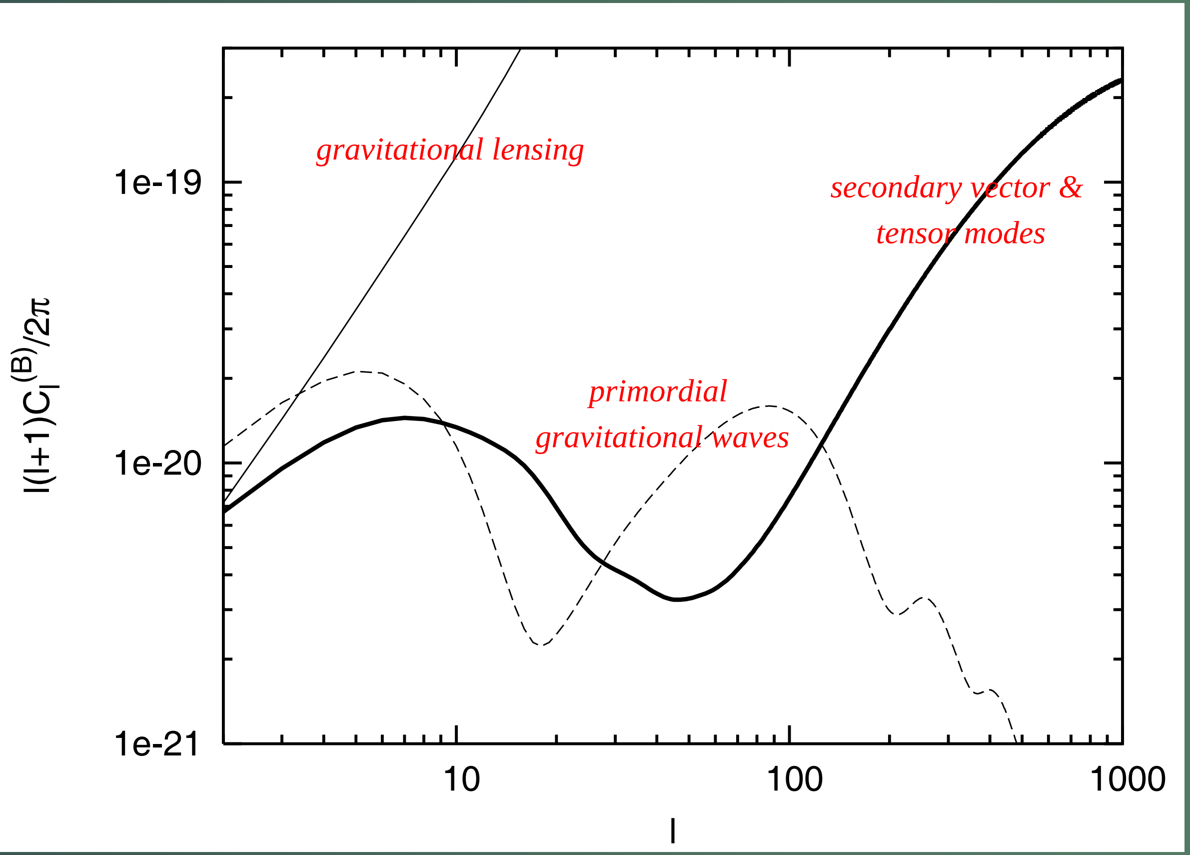
B-mode CMB polarization from scalar perturbations

Diago Harari (Buenos Aires → Bariloche)
Sabino Matarrese (Phys. Dept., Padova)
Silvia Mollerach (Centro Atomico, Bariloche)

Mollerach, Harari & Matarrese 2004, Phys. Rev. D 69 063002

Second-order effects from scalar modes & B-mode polarization

The B-mode polarization produced by primordial gravitational waves can be hidden by gravitational lensing and/or by second--order vector and tensor modes, unless the inflation energy scale is larger than 10^{15} GeV



Mollerach, Harari & Matarrese 2004, Phys. Rev. D **69** 063002

Seed magnetic fields generation from density perturbations

Sabino Matarrese (Phys. Dept., Padova)
Silvia Mollerach (Centro Atomico, Bariloche)
Alessio Notari (SNS, Pisa → McGill, Montreal)
Antonio Riotto (INFN, Padova)

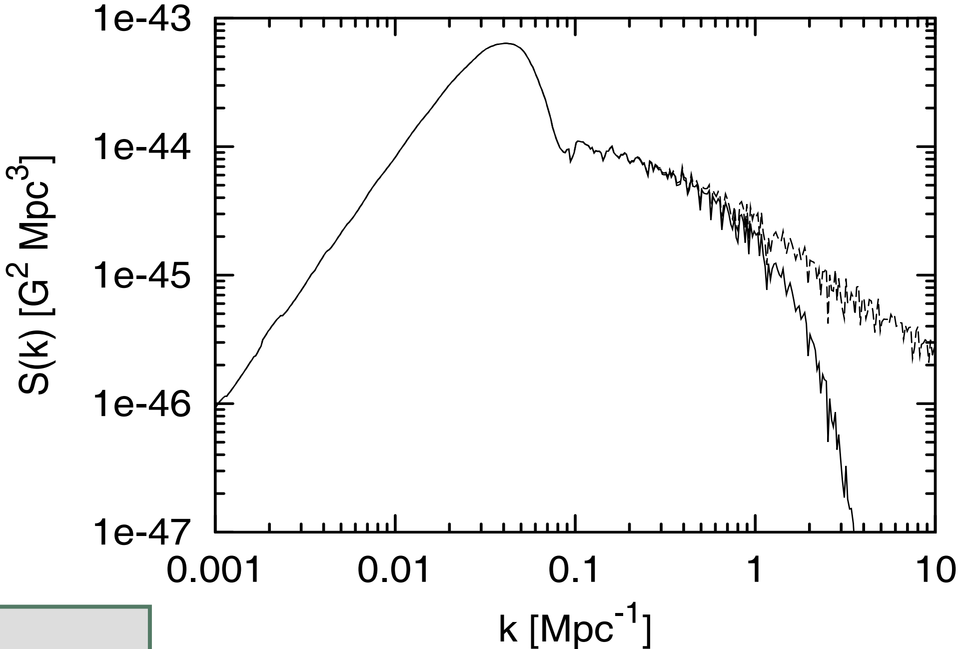
astro-ph/0410687

Magnetic field generation from secondary vector modes

We derive the *minimal* seed magnetic field which unavoidably arises in the radiation and matter eras, prior to recombination, by the rotational velocity of ions and electrons, gravitationally induced by the non-linear evolution of primordial density perturbations. The resulting magnetic field power-spectrum is fully determined by the amplitude and spectral index of density perturbations. The *rms* amplitude of the seed-field at recombination is $B \approx 10^{-23}(\lambda/\mathrm{Mpc})^{-2}$ G, on comoving scales $\lambda \gtrsim 1$ Mpc.

While the total vorticity of the system is conserved – in full agreement with Kelvin's circulation theorem – and can be set to zero, electrons and protons acquire a non-zero rotational velocity from the second-order metric vector mode, which in turn arises from the mode-mode coupling of primordial scalar modes. This rotational velocity gives rise to a seed magnetic field, according to Harrison's mechanism.

$$\mathbf{B}(\mathbf{k},\eta) = -\frac{m_p(1+z)}{e\mathcal{H}^2}\int \frac{d^3k'}{(2\pi)^3}\mathbf{k}\times\mathbf{k}' \left[2\varphi'(|\mathbf{k}-\mathbf{k}'|)\varphi(k') - \frac{k'^2}{12\mathcal{H}^2}\varphi'(|\mathbf{k}-\mathbf{k}'|)\varphi(k') - \frac{k'^2}{12\mathcal{H}}\varphi(|\mathbf{k}-\mathbf{k}'|)\varphi(k')\right],$$



Note: magnetic field generation will also arise due to recombination (Hogan 1997; Dolgov Berezhiani 2003) and reionization (Aghanim et al. 2001; Gnedin et al. 2001) physics.

Does acceleration require Dark Energy?

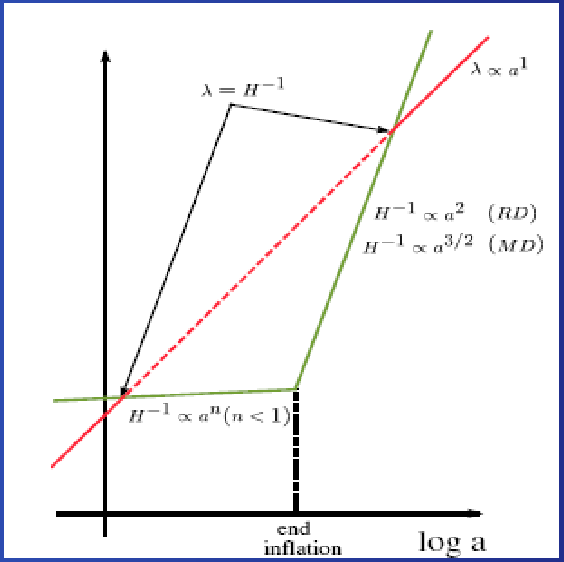
Rocky Kolb (Fermilab/Chicago)
Sabino Matarrese (Phys. Dept., Padova)
Alessio Notari (SNS, Pisa → McGill, Montreal)
Antonio Riotto (INFN, Padova)
Enrico Barausse (SISSA, Trieste)

hep-ph/0409038; astro-ph/0410541; in prep.

Solution of the horizon problem: inflation ⇐ ä > 0

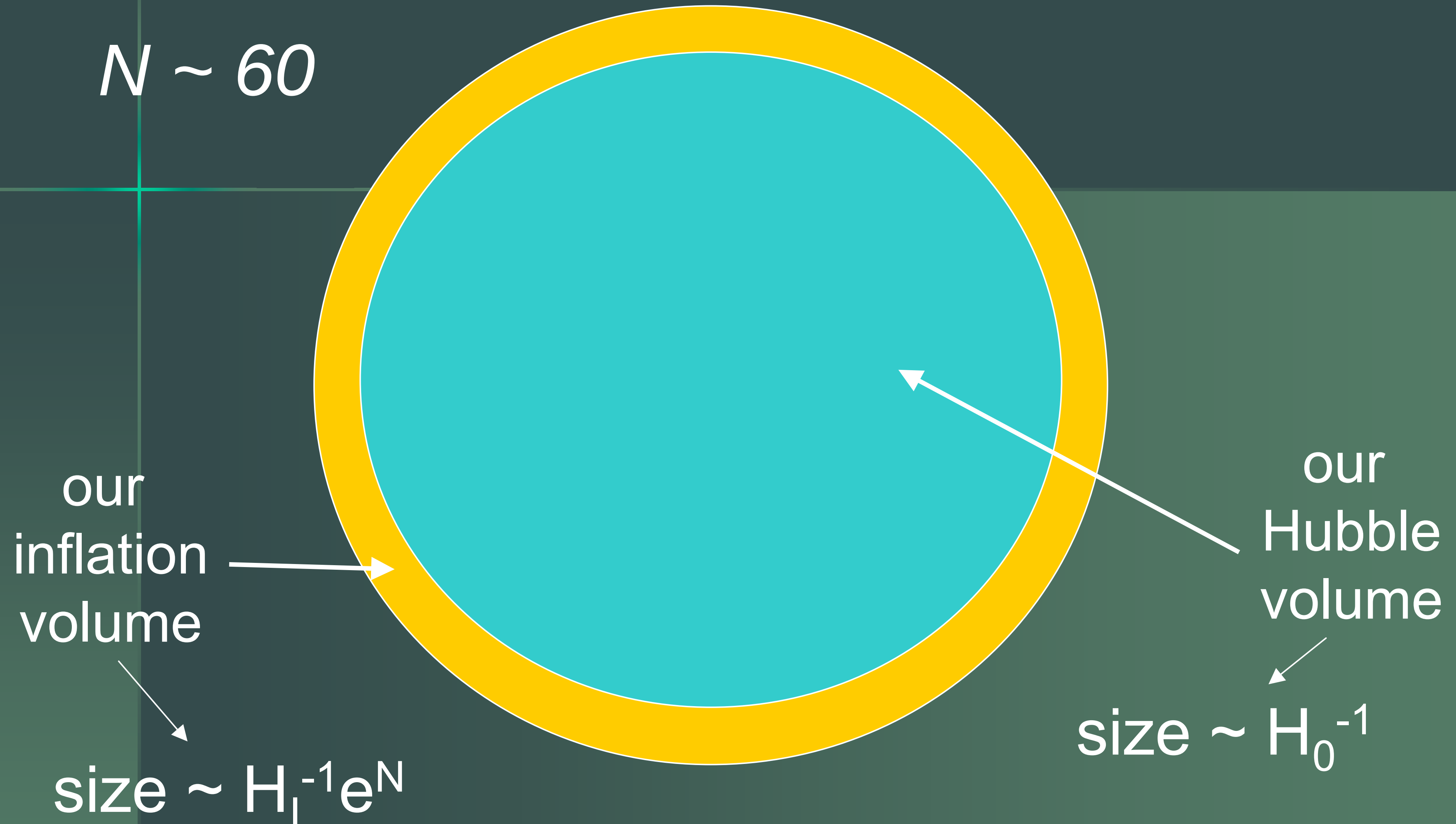
About <u>60 e-folds</u> of inflation suffice to solve the horizon and flatness problems. Inflation, however, usually lasts much much longer!

- Inflation is attained when $\frac{\ddot{a}}{a} = H^2\left(\frac{\dot{H}}{H^2}+1\right) > 0$
- If during inflation the Universe suffers a quasi-de Sitter phase $\dot{H} \approx 0$ and $H^2 \approx$ constant
- The scale factor grows exponentially,
  $a(t) = a\left(t_\star\right)\, e^{\int_{t_\star}^{t} H(t')\, dt'} \approx a\left(t_\star\right)\, e^N$
- $N = \int_{t_{\rm BI}}^{t_{\rm EI}} H(t')\, dt' \approx H\left(t_{\rm EI} - t_{\rm BI}\right) =$ number of e-foldings

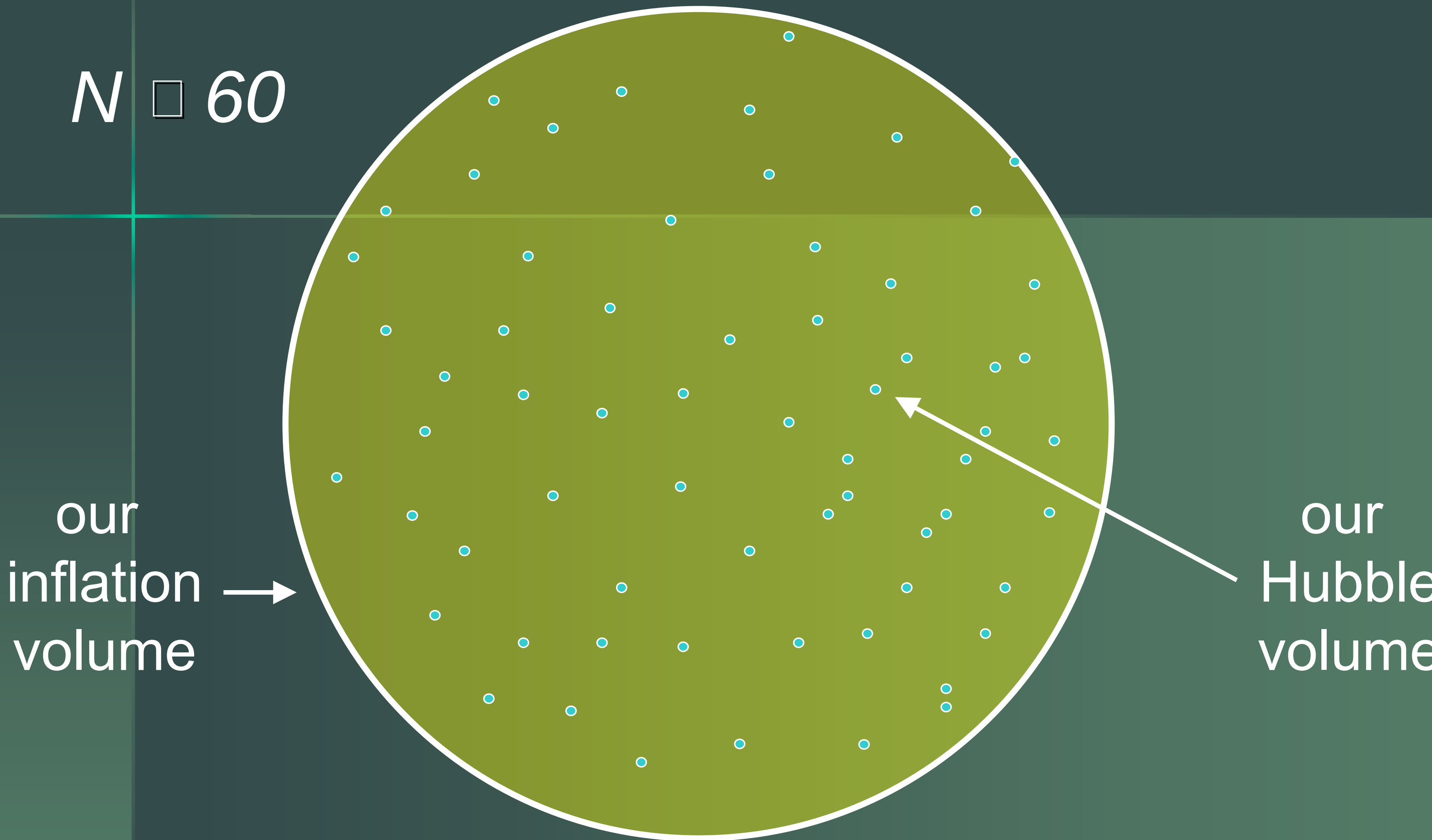


$$\frac{d}{dt}\left(\frac{\lambda}{R_H}\right) > 0 \Rightarrow \ddot{a} > 0$$

N ~ 60



N □ *60*

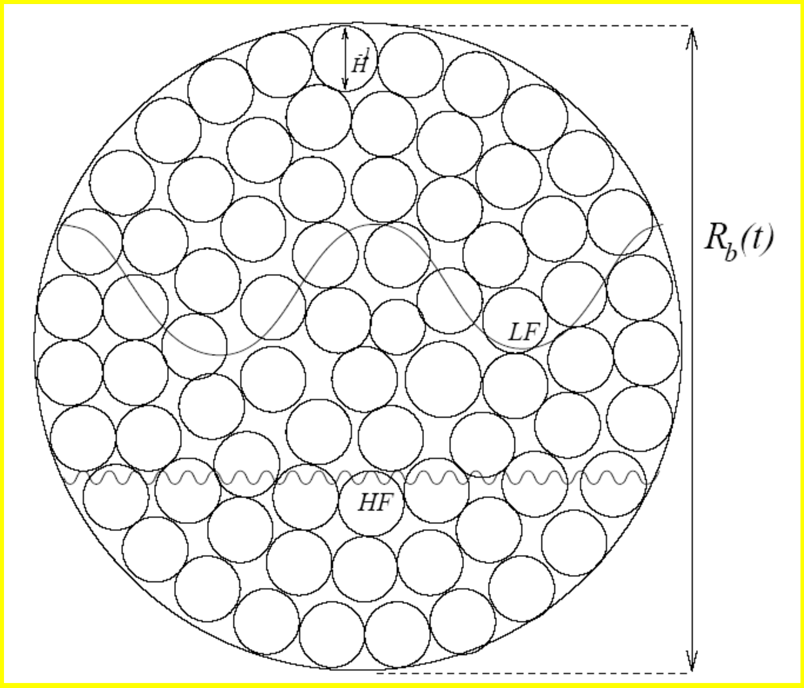


Crucial point

- *Remember the statistical nature of the cosmological fluctuations induced by quantum vacuum oscillations of the inflaton*
- *The observable Universe corresponds to just one (possibly typical) member of an ensemble of possible Universes*
- *Our observed Universe is only a tiny fraction of the entire inflated Universe*

Do super-Hubble perturbations have an impact on physical observables?

- *Long wavelength super-Hubble perturbations do exist*
- *They will cross the Hubble radius in the far future*

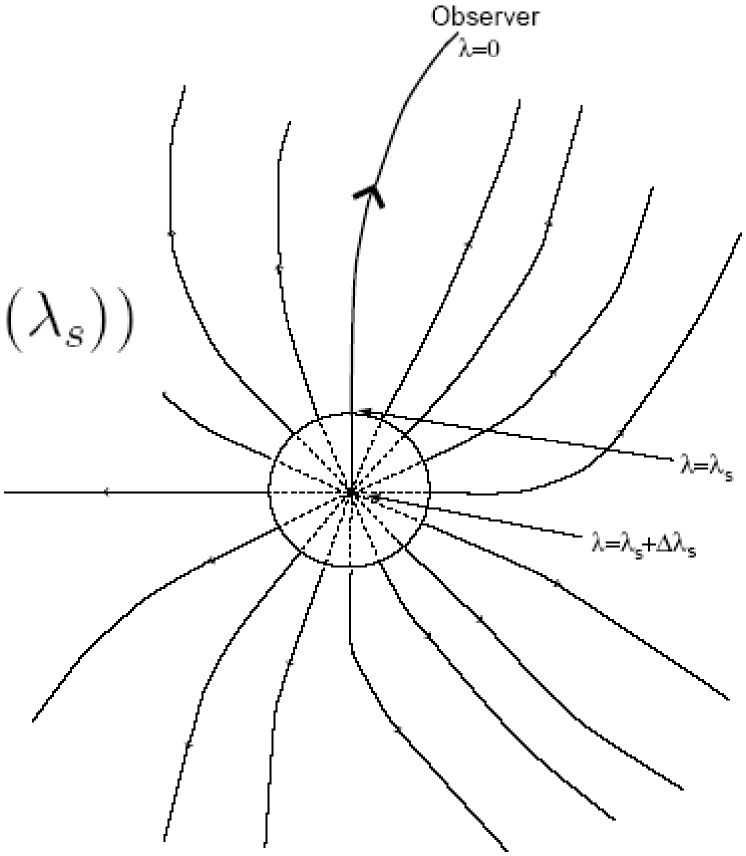


Luminosity distance-redshift relation in a perturbed Universe

$$L = 4\pi R^2 \ell(\lambda_s)$$

$$d_L = R\sqrt{\frac{\ell(\lambda_s)}{\ell(0)}} = R\frac{A(\lambda_s)}{A(0)}(1 + z(\lambda_s))$$

$$1 + z(\lambda_s) := \frac{\omega(\lambda_s)}{\omega(0)}$$



Second-order computation

$$g_{\mu\nu} = g_{(0)\mu\nu} + g_{(1)\mu\nu} + g_{(2)\mu\nu} \qquad g_{(0)\mu\nu} = \eta_{\mu\nu}$$

$$g_{(1)0j} = 0, \quad g_{(1)00} = 0 \qquad g_{(2)0j} = 0, \quad g_{(2)00} = 0$$

$$g_{(2)ij} =$$

$$\left(1 - 2\psi^{(1)} - \psi^{(2)}\right)\delta_{ij} + D_{ij}\left(\chi^{(1)} + \frac{1}{2}\chi^{(2)}\right) + \frac{1}{2}\left(\partial_i\chi_j^{(2)} + \partial_j\chi_i^{(2)} + \chi_{ij}^{(2)}\right)$$

synchronous and comoving gauge

Metric up to second-order

$$\chi_{(1)ij} = -\frac{\eta^2}{3}\left(\varphi_{,ij} - \frac{1}{3}\delta_{ij}\nabla^2\varphi\right) ,$$

$$\phi_{(1)} = \frac{5}{3}\varphi + \frac{\eta^2}{18}\nabla^2\varphi ,$$

φ = *linear peculiar gravitational potential produced by inflaton fluctuations*

$$\phi_{(2)} = -\frac{50}{9}\varphi^2 - \frac{5\eta^2}{54}\varphi^{,k}\varphi_{,k} + \frac{\eta^4}{252}\left(-\frac{10}{3}\varphi^{,ik}\varphi_{,ik} + \left(\nabla^2\varphi\right)^2\right) ,$$

$$\chi^{(2)}_{ij} = \frac{\eta^4}{126}\left(19\varphi^{,k}{}_{,i}\varphi_{,kj} - 12\varphi_{,ij}\nabla^2\varphi + 4(\nabla^2\varphi)^2\delta_{ij} - \frac{19}{3}\varphi^{,kl}\varphi_{,kl}\delta_{ij}\right) - \frac{10\eta^2}{9}\left(\varphi_{,i}\varphi_{,j} - \frac{1}{3}\varphi^{,k}\varphi_{,k}\delta_{ij}\right) + \chi^{(2)\perp}_{ij} ,$$

2-nd order tensors

Luminosity distance-redshift relation in a perturbed Universe

$$d_L = \frac{c}{H_p}\left[z + \frac{z^2}{2}\left(1 - q_p\right) + ... \right]$$

$$H_p = H_0\left(1 - \frac{10}{27}\varphi\nabla^2\varphi + ... \right)\frac{a}{a_0}$$

$$q_p = q_0\left(1 + \frac{50}{27}\varphi\nabla^2\varphi + ... \right)\frac{a}{a_0}$$

Variance of q_0

are super–Hubble perturbations physical?

constant φ can be scaled out of equations

... but can't get rid of $\varphi \nabla^2 \varphi$!

$$\mathrm{Var}\left[\varphi\nabla^2\varphi\right] \simeq \left(\frac{9}{4}a_0^4 H_0^4\right)^2 \int \frac{dk_1}{k_1}\Delta^2(k_1,a_0)\int \frac{dk_2}{k_2^5}\Delta^2(k_2,a_0)$$

for a scale-invariant spectrum, n=1

$$\frac{\sqrt{\mathrm{Var}\left[\langle\widetilde{q}\rangle_\Omega\right]}}{q_0} \simeq 10^{-10}\ln\frac{k_{\mathrm{MAX}}}{k_{\mathrm{MIN}}} \simeq 1 \qquad (10^{18.8} \quad e\text{-folds!})$$

$$\Delta^2(k) \propto k^{3+n} \text{ with } 0 < (1-n) \ll 1$$

$$n = 0.94 \qquad \sim 700 \ e\text{-folds}$$

The variance of q_0 can be large

$$\sqrt{Var(q)} \approx q_0 \sqrt{Var(\varphi \nabla^2 \varphi)} \frac{a}{a_0} \approx q_0$$

$$N_{\text{TOTAL}} - N_H \geq 10^{18.8}$$ → *Harrison–Zel'dovich*

$$N_{\text{TOTAL}} - N_H \geq 676$$ → $n = 0.94$

Variance of q_0

- What we really want is q_0 in our Hubble volume

our inflation volume →

infrared cutoff at $k = k_{\mathrm{MIN}}$

k_{MIN} related to # of inflation e-folds

$$\ln\left(\frac{k_H}{k_{\mathrm{MIN}}}\right) = N_{\mathrm{TOTAL}} - N_H$$

our Hubble volume

Important technical issues

$$\frac{\delta\rho}{\rho} \propto \nabla^2\varphi$$

→ *on super-horizon scales density perturbations are < 1*

$$\varphi\nabla^2\varphi$$ *comes from* $$e^{-\varphi}\nabla^2\varphi$$

we may easily go beyond second order

Local conclusions on back-reaction

- *The Hubble parameter and the deceleration parameter are not deterministic*
- *Because of the statistical nature of vacuum fluctuations, the gravitational potential does not have a well-defined value*
- *The theoretical predictions of the background cosmological parameters come with a non-vanishing cosmic variance implying an intrinsic theoretical error*
- *The real, i.e. perturbed, Universe does not require dark energy to drive acceleration in the observable Hubble-size patch of the global inflated region.*
- *Inflation might have lasted enough to solve the horizon and flatness problems, but also to explain present-day acceleration.*

Global conclusions

- *relativistic second-order cosmological perturbation theory, despite its technical complexities, allows to describes a variety of novel phenomena which are either totally absent or poorly represented by the conventional linear perturbative approach*

- *... more surprises ... to come!*